# MATH 502: REAL AND COMPLEX ANALYSIS

SPRING 2004

**A.Katok**

PROBLEMS ON BANACH SPACES:

F1. Consider the linear space of all polynomials with real or complex coefficients. Prove that it s impossible to introduce a norm in this space which makes it a Banach space.

F2. Prove that Lebesgue measure on the circle $\mathbb{R}/\mathbb{Z}$ can be extended to a *finitely additive* translation invariant measure defined on all subsets of $\mathbb{R}/\mathbb{Z}$.

*Hint:* When you use the Hahn-Banach Theorem you need to check that if $f$ is a bounded Lebesgue measurable function and for some $t \in \mathbb{R}/\mathbb{Z}$, $f(x) = g(x+t) - g(x)$ where $g$ is a bounded function, then there is also a Lebesgue measurable function $h$ such that $f(x) = h(x+t) - h(x)$.

F3. If the closed unit ball in a Banach space is compact the space is finite–dimesnsional.

*Hint:* Show that for aclosed subspace $H$ in Banach space $B$ one can find $v \in B$ such that $|v\| = 1$ and $\|v - u\| > 1/2$ for every $u \in H$ (in fact, one can replace $1/2$ by any fixed number $\alpha < 1$).

F4. Prove that the dual space to every normed space is complete.

F5. The *weak toplogy* in a normed space is defined as the weakest topology in which all bounded linear functionals are continuous. Prove that every weakly converving sequence is bounded in norm.

F6. Prove that if the dual space to a Banach space is strictly convex (i.e. its unit ball is strictly convex) then norm–preserving extension of a any linear function from any subspace to the whole space is unique.