

Name: \_\_\_\_\_ ID: \_\_\_\_\_

## Math 251

### Quiz 2

There are SIX problems(5 points each), of which the highest FIVE will constitute your grade.

1. Determine the longest interval in which the given initial value problem has a unique solution.

$$(x - 3)y'' + xy' + (\ln |x|)y = 0, \quad y(1) = 0, \quad y'(1) = 1.$$

Key:  $(0, 3)$ .

2. Find the Wronskian of two solutions of the following equation. (You may not want to try to solve the equation)

$$t^2y'' - t(t + 2)y' + (t + 2)y = 0.$$

Key: (By Abel Theorem)  $ct^2e^t$ .

3. Solve the I.V.P:

$$y'' + 4y' + 5y = 0 \quad y(0) = 1, \quad y'(0) = 0.$$

Key:  $y = e^{-2t} \cos t + 2e^{-2t} \sin t.$

4. Solve the I.V.P:

$$9y'' - 12y' + 4y = 0, \quad y(0) = 2, \quad y'(0) = -1.$$

Key:  $y = 2e^{\frac{2}{3}t} - \frac{7}{3}te^{\frac{2}{3}t}.$

5. Find the complementary solution and particular solution to the following differential equation:

$$y'' - 2y' - 3y = 3te^{2t}.$$

Key:  $y = C_1e^{3t} + C_2e^{-t} + (-t - \frac{2}{3})e^{2t}$ .

6. A 2kg object attached to a spring which is stretched by  $\frac{1}{40}$ m. The object has the initial displacement of 1m downwards, with the initial downward velocity 3m/sec. Suppose there are no other forces except gravity and spring force. Find the displacement of the object at any time.

Key:  $u(t) = \cos(20t) + \frac{3}{20} \sin 20t$ .