

Name: _____ ID: _____

Math 251

Quiz 1

There are SIX problems(5 points each), of which the highest FIVE will constitute your grade.

1. Solve the following initial value problem by using method of integrating factor:

$$y' + \frac{2}{t}y = \frac{\cos t}{t^2}, \quad y(\pi) = 0, t > 0.$$

Key: $y = \frac{\sin t}{t^2}$.

2. Determine the maximal interval in which the solution to the given initial value problem exists.

$$(4 - t^2)y' - ty = 2t^3, \quad y(1) = 5.$$

Key: $(-2, 2)$.

3. Find the equilibrium solutions to the following equation and study the stability of them.

$$y' = (2 - e^y)(y + 1).$$

Key: $y = \ln 2$ stable; $y = -1$ unstable.

4. Show that the following equation is exact, and find the solution to it.

$$(2xy^2 + 2y) + (2x^2y + 2x)y' = 0.$$

Key: $x^2y^2 + 2xy + C = 0$.

5. A swimming pool is initially filled with $500m^3$ of water with a chlorine concentration of $10\frac{g}{m^3}$. Water containing $50\frac{g}{m^3}$ of chlorine flows into the pool at a rate of $5m^3$ per minute. The well-mixed water in the pool is drained away at the same rate. Find the amount of chlorine in the pool at any time $t > 0$.

Key: $Q(t) = 25000 - 20000e^{-t/100}$.

6. Solve the following second-order linear equation:

$$y'' + 4y' + 3y = 0, \quad y(0) = 2, \quad y'(0) = -1.$$

Key: $y(t) = \frac{5}{2}e^{-t} - \frac{1}{2}e^{-3t}$.