Key to Exam I:

1. C
2. C
3. B
4. D
5. Implicit form is $y^2 + 4y - 2(x^3 + \cos x - 1) = 0$;

   Explicit form is $y = -2 - \sqrt{2x^3 + 2\cos x + 2}$.
6. (a) $Q'(t) = 20 - \frac{2Q(t)}{80-t}$, $Q(0) = 0$, $0 < t < 80$

   (b) Integrating factor is $\frac{1}{(80-t)^2}$, and $Q(t) = -\frac{1}{4}t^2 + 20t$.
7. (a) $-2, \frac{9}{2}, 8$.

   (b) -2 and 8 are unstable solutions; $\frac{9}{2}$ is a stable solution.

   (c) $\frac{9}{2}$.

   (d) 8.
8. (a) It is easy to verify that $M_y = N_x$.

   (b) $y \sin \pi x + x^3 y - 2e^x + 5y = -13 - 2e^2$.
9. (a) $y(t) = 3e^{4t} - 2te^{4t}$.

   (b) $3 - 2t$ goes to $-\infty$ and e^{4t} goes to ∞, so the product of them will go to $-\infty$.
10. Let $y_2(t) = t^2 v(t)$, we can find $v''t + v' = 0$. Let $u = v'$, then $u = \frac{1}{t}$, so $v = \ln t$. Therefore $y_2(t) = t^2 \ln t$, and the general solution will be $y(t) = C_1 t^2 + C_2 t^2 \ln t$.

11. (a) $y_c = C_1 e^{2t} + C_2 e^{-t}$.

(b) Make an initial guess $y_p = A\cos 2t + B\sin 2t$. One finds that $A = \frac{3}{20}, B = -\frac{9}{20}$.

(c) $y_p = (At^3 + Bt^2 + Ct + D)te^{-t} + Ee^{2t}\cos 6t + Fe^{2t}\sin 6t$.