Math 22 Section 9

Practice for Quiz 2 Solutions

1. Write the following in the form $a + bi$:

   (a) $(2 - 7i)(1 + 3i)$

   $(2-7i)(1+3i) = 2+6i-7i-21i^2 = 2-i+21 = 23-i$

   (b) $\frac{2+3i}{1-4i}$

   $\frac{2+3i}{1-4i} = \frac{2+3i}{1-4i}\frac{1+4i}{1+4i} = \frac{2+8i+3i+12i^2}{1^2+4^2} = \frac{2+11i-12}{17} = \frac{-10+11i}{17}$
   $= \frac{-10}{17} + \frac{11}{17}i$

2. Find the distance and midpoint between the points $(3, 5)$ and $(-5, 2)$.

   $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} = \sqrt{(-5-3)^2 + (2-5)^2} = \sqrt{(-8)^2 + (-3)^2} = \sqrt{64+9} = \sqrt{73}$

   $midpoint = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right) = \left(\frac{3-5}{2}, \frac{5+2}{2}\right) = \left(-1, \frac{7}{2}\right)$

3. Solve the inequality. Express the solution as an interval.

$$\frac{x-2}{x+5} \leq 2$$

This is equivalent to $\frac{x-2}{x+5} - 2 \leq 0$

or $\frac{x-2}{x+5} + \frac{-2(x+5)}{x+5} = \frac{x-2-2x-10}{x+5} = \frac{-x-12}{x+5} \leq 0$

or $\frac{x+12}{x+5} \geq 0$

Since the numerator is 0 at $x = -12$ and the denominator is 0 at $x = -5$ we construct the table:

	$(-\infty, -12)$	$(-12, -5)$	$(-5, \infty)$
$x + 12$	-	+	+
$x + 5$	-	-	+
$\frac{x+12}{x+5}$	+	-	+

since $x = -12$ is a zero and the function is undefined at $x = -5$, we include the former in our intervals but not the latter. The two intervals for which the inequality holds are thus $(-\infty, -12]$ and $(-5, \infty)$.

4. An old scale measures weight with an accuracy of $\pm$1lb. If a merchant sells potatoes using this scale at \$2/lb, by how much could he be overcharging or undercharging a customer who buys 10lb of potatoes?

   There are two ways to solve this:

   (a) Let x be the amount of possible over/undercharging.
   The weight the scale will read will be 10 ± 1lb
   The cost should be 10lb$\times$\$2/lb=\$20
   The amount the customer is charged will be as high as \$$(10 + 1) \times 2 =$\$22
   or as low as \$$(10 - 1) \times 2 =$\$18
   The relations are:
   high price = correct price + amount of overcharging
   low price = correct price - amount of undercharging
   Plugging in from above,
   $22 = 20 + x$
   $18 = 20 - x$
   so $x = 2$ in both cases.
   So, the customer could be overcharged or undercharged by as much as \$2.

   (b) Alternatively, an astute observer will note that the amount of undercharging or overcharging does not depend on the weight of the potatoes being bought.
   Since the scale has an uncertainty of $\pm$1lb, we can multiply this by the cost per pound to get the uncertainty in the price. $\pm$1lb$\times$\$2/lb= $\pm$\$2. So, the customer could be overcharged or undercharged by as much as \$2.

5. Find all solutions to

$$x^{\frac{1}{2}} - 4x^{\frac{1}{4}} + 3 = 0$$

Making the substitution $w = x^{\frac{1}{4}}$, we get the quadratic equation

$$w^2 - 4w + 3 = 0$$

This can be factored easily and is equivalent to

$$(w-3)(w-1) = 0$$

So $w = 3$ or $w = 1$

The first case implies $x^{\frac{1}{4}} = 3$ or $x = 3^4 = (3^2)^2 = 9^2 = 81$

The second case implies $x^{\frac{1}{4}} = 1$ or $x = 1^4 = 1$

So the two possible solutions are $x = 1$ and $x = 81$

6. Solve the inequality and express the solution as an interval:

$$\frac{1}{|x+5|} \geq 2$$

First, note that the LHS is undefined for $x = -5$.

The inequality is then equivalent to $1 \geq 2|x+5|$, $x \neq -5$

or $\frac{1}{2} \geq |x+5|$, $x \neq -5$

or $-\frac{1}{2} \leq x + 5 \leq \frac{1}{2}$, $x \neq -5$

or $-\frac{1}{2} - 5 \leq x \leq \frac{1}{2} - 5$, $x \neq -5$

or $-\frac{11}{2} \leq x \leq -\frac{9}{2}$, $x \neq -5$