The examination consists of 20 multiple choice questions. For each problem, please fill in the bubble on the scantron sheet and circle the correct answer on your examination. Each problem is worth five points.

THE USE OF CALCULATORS IS NOT PERMITTED IN THIS EXAMINATION.

CHECK THE EXAMINATION BOOKLET BEFORE YOU START. THERE SHOULD BE 20 PROBLEMS ON 11 PAGES (INCLUDING THIS ONE).
1. Let \( f(x) = 2x^3 - 3x^2 - 12x + 5 \). Find the intervals where \( f \) is increasing and intervals where \( f \) is decreasing.

   a) \( f \) is increasing on \((-1, 2)\); decreasing on \((\infty, -1)\) and on \((2, \infty)\).

   b) \( f \) is increasing on \((-2, 3)\); decreasing on \((\infty, -2)\) and on \((3, \infty)\).

   c) \( f \) is increasing on \((\infty, -3)\); decreasing on \((3, \infty)\).

   d) \( f \) is increasing on \((\infty, -1)\) and on \((2, \infty)\); decreasing on \((-1, 2)\).

2. Suppose the first derivative of \( f(x) \) is given by

\[
\frac{d}{dx} f(x) = \frac{x(x - 1)(x + 2)^2}{x^2 + 1}.
\]

Find the relative maxima and relative minima of \( f \).

   a) relative maximum at \( x = 0 \); relative minimum at \( x = -1 \) and at \( x = -2 \).

   b) no relative maxima or minima.

   c) relative maximum at \( x = 0 \); relative minimum at \( x = 1 \).

   d) relative maximum at \( x = 1 \) and at \( x = -2 \); relative minimum at \( x = 0 \).
3. Suppose the second derivative of \( f(x) \) is given by
\[
\frac{d^2}{dx^2} f(x) = \frac{2x^2 - 10x}{(x^2 + 1)^3}.
\]

Determine the intervals of concavity of \( f \).

a) \( f \) is concave down on \((-\infty, 5)\); concave up on \((5, \infty)\).

b) \( f \) is concave up on \((-\infty, 0)\) and on \((5, \infty)\); concave down on \((0, 5)\).

c) \( f \) is concave down on \((-\infty, 0)\); concave up on \((0, 5)\) and on \((5, \infty)\).

d) \( f \) is concave up on \((-\infty, -1)\) and on \((5, \infty)\); concave down on \((-1, 5)\).

4. Determine all vertical and horizontal asymptotes of the function
\[
g(x) = \frac{3x(x - 2)}{x^2 - x - 20}.
\]

a) vertical asymptotes \( x = 5 \) and \( x = -4 \); horizontal asymptotes \( y = 0 \) and \( y = 2 \).

b) vertical asymptotes \( x = 5 \) and \( x = -4 \); horizontal asymptote \( y = 3 \).

c) vertical asymptotes \( x = 0 \) and \( x = 2 \); no horizontal asymptote.

d) no vertical asymptote; horizontal asymptote \( y = \frac{3}{10} \).
5. A company’s cost (in dollars) for producing \( x \) units of their product is

\[
C(x) = 0.01x^2 + 5x + 49, \quad (0 < x < \infty).
\]

Determine the average cost function, \( \bar{C}(x) \), and determine the value of \( x \) that minimizes \( \bar{C}(x) \).

a) \( \bar{C}(x) \) is minimum when \( x = 7 \).

b) \( \bar{C}(x) \) is minimum when \( x = 250 \).

c) \( \bar{C}(x) \) is minimum when \( x = 70 \).

d) \( \bar{C}(x) \) is minimum when \( x = -250 \).

6. Find the absolute maximum and absolute minimum values of the function

\[
f(x) = x^4 - 2x^2 + 5 \quad \text{on} \quad [0, 2].
\]

a) absolute maximum is 20; absolute minimum is 5.

b) absolute maximum is 13; absolute minimum is -2.

c) absolute maximum is 5; absolute minimum is 4.

d) absolute maximum is 13; absolute minimum is 4.
7. What is the maximum possible value of

\[ A = (x - 1)(y - 2), \]

if \( x > 0 \) and \( xy = 50 \).

a) 25  
b) \frac{65}{2}  
c) 32  
d) \frac{95}{3}

8. A rectangular box with square base and no top must hold 4 cubic feet. What dimensions minimize the total surface area (the base plus the four sides)?

a) \( \frac{3}{4} \times \frac{3}{4} \times \frac{64}{9} \)  
b) 1 \times 1 \times 4  
c) \( \frac{3}{2} \times \frac{3}{2} \times \frac{16}{9} \)  
d) 2 \times 2 \times 1
9. Simplify \( A = \log_3 27 + \ln \left( \frac{1}{e^3} \right) - \log_5 1 \).

   a) \( A = -2 \)
   b) \( A = 0 \)
   c) \( A = 1 \)
   d) \( A = 5 \)

10. If \( 3^{t-1} = 9^t \), what is \( t \)?

    a) \( t = 2 \)
    b) \( t = -2 \)
    c) \( t = -1 \)
    d) \( t = 0 \)
11. Given that $\ln 2 \approx 0.7$, and $\ln 3 \approx 1.1$ what is $\ln \sqrt{6}$?

a) $\ln \sqrt{6} \approx 0.9$

b) $\ln \sqrt{6} \approx 1.2$

c) $\ln \sqrt{6} \approx 0.75$

d) $\ln \sqrt{6} \approx 1.1$

12. If $1000$ is invested at $3.6\%$ per year compounded *monthly*, what will be the accumulated amount after 8 years?

a) $1000e^{3.6}$ dollars.

b) $1000(1.036)^8$ dollars.

c) $1000(1.003)^{96}$ dollars.

d) $1000(1.036)^{96}$ dollars.
13. What amount must be invested now at 6% per year compounded continuously to yield $10,000 in 5 years?

a) \( \frac{10,000}{e^{0.3}} \) dollars

b) \( 10,000e^{-0.06} \) dollars

c) \( 10,000e^{0.3} \) dollars

d) \( \frac{10,000}{(1 + e^{0.06})^5} \) dollars

14. What is the present value \( P \) of \( A = $1000 \) two years from now if the interest rate is 12% compounded monthly?

a) \( P = 1000(1.1)^{24} \)

b) \( P = 1000(1.01)^{-24} \)

c) \( P = \frac{1000}{1.24} \)

d) \( P = 1000(1.12)^{-24} \)
15. If interest is paid at 6% compounded continuously, what is the effective interest rate $r_{\text{eff}}$?

a) $r_{\text{eff}} = e^{1.06}$.

b) $r_{\text{eff}} = e^{0.06}$.

c) $r_{\text{eff}} = e^{0.06} - 1$.

d) $r_{\text{eff}} = 1 - e^{1.06}$.

16. Find $\frac{d}{dt} ((t^2 + 1)e^{3t})$.

a) $6t(t^2 + 1)e^{3t} - 1$

b) $2t + 3e^{3t}$

c) $(3t^2 + 2t)e^{3t}$

d) $(3t^2 + 2t + 3)e^{3t}$
17. If \( f(x) = \ln(x^2 - 3x + 4) \), find \( f'(x) \).

\[
\begin{align*}
a) & \quad \ln(2x - 3) \\
b) & \quad \frac{2x - 3}{x^2 - 3x + 4} \\
c) & \quad \frac{1}{x^2 - 3x + 4} \\
d) & \quad \frac{\ln(x^2 - 3x + 4)}{2x - 3}
\end{align*}
\]

18. Find \( \frac{d^2}{dt^2} (te^t) \).

\[
\begin{align*}
a) & \quad (2 + t)e^t \\
b) & \quad 1 + te^t \\
c) & \quad 3te^{t-1} + t^3e^{t-2} \\
d) & \quad e^t + t^2e^{t-1}
\end{align*}
\]
19. Find $\frac{d^2}{dx^2} (\ln x)^2$

   a) $\frac{2 - 2 \ln x}{x^2}$
   b) $\frac{2}{x}$
   c) $\frac{2 \ln x}{x}$
   d) $x - \frac{2}{x^2}$

20. Use logarithmic differentiation to find $\frac{d}{dx} \left( \frac{(x + 1)^5}{\sqrt{x^2 - 1}} \right)$.

   a) $\frac{(x + 1)^5}{\sqrt{x^2 - 1}} \left[ \frac{5}{x + 1} - \frac{x}{x^2 - 1} \right]$
   b) $5x(x + 1)^4 \sqrt{x^2 - 1}$
   c) $\frac{5(x + 1)^4}{\sqrt{x^2 - 1}} + \frac{(x + 1)^5}{(x^2 - 1)^{\frac{3}{2}}}$
   d) $\frac{(x + 1)^5}{\sqrt{x^2 - 1}} \left[ 5 \ln(x + 1) - \frac{1}{2} \ln(x^2 - 1) \right]$
21. KEY: 1-d, 2-c, 3-b, 4-b, 5-c, 6-d, 7-c, 8-d, 9-b, 10-c, 11-a, 12-c, 13-a, 14-b, 15-c, 16-d, 17-b, 18-a, 19-a, 20-a.