MATH 110 NAME _____

FINAL EXAMINATION STUDENT NUMBER _____

MAY 3, 2005 INSTRUCTOR _____

TEST FORM A SECTION NUMBER _____

This examination will be machine processed by University Testing Services. Use only a number 2 pencil on your answer sheet. On your answer sheet identify your name, this course (MATH 110) and the date. Code and blacken the corresponding circles on your answer sheet for your student I.D. number and class section number. Code in your test form.

There are **30** multiple choice questions. For each problem **four** possible answers are given, only one of which is correct. You should solve the problem, note the letter of the answer that you wish to give and **blacken** the corresponding space on the **answer sheet**. Mark only one choice; darken the circle completely (you should not be able to see the letter after you have darkened the circle). Check frequently to be sure the problem number on the test sheet is the same as the problem number of the answer sheet.

THE USE OF CALCULATORS AND OTHER PORTABLE ELECTRONIC DEVICES IS NOT PERMITTED DURING THIS EXAMINATION.

THE USE OF NOTES OF ANY KIND IS NOT PERMITTED DURING THIS EXAMINATION.

1. Which of the following is the inverse of the function $f(x) = 2x + 3$?

   a) $f^{-1}(x) = 2x - 3$

   b) $f^{-1}(x) = -2x - 3$

   c) $f^{-1}(x) = \dfrac{x - 3}{2}$

   d) $f^{-1}(x) = \dfrac{2}{x} + 3$

2. Find

$$A = \lim_{x \to 2} \frac{x^2 + 2x - 8}{x^2 - 3x + 2}$$

   and

$$B = \lim_{x \to \infty} \frac{10x^5 + 7x^3 + x}{5x^6 + 8x - x^2}.$$

   a) A does not exist, $B = 0$

   b) $A = 1$, B does not exist

   c) $A = 0$, $B = 2$

   d) $A = 6$, $B = 0$

3. Let

$$f(x) = \begin{cases} 3x - 4 & \text{if } x \leq 2 \\ x^2 + kx & \text{if } x > 2. \end{cases}$$

Find the value of k for which $f(x)$ is continuous at $x = 2$.

a) 2

b) $\sqrt{2}$

c) -1

d) 1

4. Find

$$\lim_{h \to 0} \frac{(3+h)^2 - 3^2}{h}.$$

a) 6

b) 16

c) 8

d) 4

5. Let $f(s) = \dfrac{1-2s}{1+3s}$. Find $f'(s)$.

   a) $\dfrac{-5+12s}{(1+3s)^2}$

   b) $\dfrac{-5}{(1+3s)^2}$

   c) $\dfrac{5-12s}{(1+3s)^2}$

   d) $\dfrac{12s}{(1+3s)^2}$

6. Find the equation of the line tangent to the graph of $f(x) = (x^2+1)(x-3)$ at the point $(0,-3)$.

   a) $y = -x - 3$

   b) $y = x - 3$

   c) $y = -3x - 3$

   d) $y = -3x$

7. The height of a projectile at time t is given by $h(t) = 120t^2 - 5t^4$. Find the maximum **velocity** of the projectile.

   a) 300

   b) 320

   c) 340

   d) 360

8. The circle of radius 10 centered at the origin satisfies the equation $x^2 + y^2 = 100$. Find the slope of the tangent line to the graph of this circle at the point $(8, 6)$.

   a) 1

   b) $-\frac{4}{3}$

   c) $\frac{3}{4}$

   d) 0

9. A 13ft ladder is sliding down a vertical wall at a rate of -0.5 feet/second. Find the rate at which the bottom of the ladder is sliding away from the wall when the top of the ladder is 5ft above the ground.

   a) $\frac{1}{2}$

   b) $\frac{5}{13}$

   c) $\frac{13}{12}$

   d) $\frac{5}{24}$

10. Let $f(x) = x^2 e^x$. Find the intervals where f is increasing and where f is decreasing.

   a) Increasing on $(-\infty, -2)$, decreasing elsewhere.

   b) Increasing on $(0, \infty)$, decreasing elsewhere.

   c) Increasing on $(-2, 0)$, decreasing elsewhere.

   d) Increasing on $(-\infty, -2)$ and $(0, \infty)$, decreasing elsewhere.

11. Determine all vertical and horizontal asymptotes of the function

$$g(x) = \frac{x^4 + 5x + 1}{x^2 - 4}.$$

a) vertical asymptotes $x = 1$ and $x = -1$; horizontal asymptote at $y = 1$.

b) vertical asymptotes $x = 1$ and $x = -1$; horizontal asymptote $y = 0$.

c) vertical asymptotes $x = -2$ and $x = 2$; horizontal asymptote at $y = 2$.

d) vertical asymptotes $x = -2$ and $x = 2$; no horizontal asymptote.

12. Find the absolute maxima and absolute minima, if any, of the function $f(x) = -x^3 + 3x$ on $[0, 3]$.

a) absolute maximum is 2, absolute minimum is -18

b) absolute maximum is 1, absolute minimum is -18

c) absolute maximum is 2, absolute minimum is -2

d) absolute maximum is 1, absolute minimum is -2

13. The daily cost function (in dollars) for producing x units of a certain product is $C(x) = \frac{1}{3}x^3 - x^2 - 3x + 200$. What is the smallest daily cost?

    a) \$171

    b) \$191

    c) \$211

    d) \$231

14. A box with square bottom and top is to have surface area of $600cm^2$. Find the dimensions of the box of largest volume.

    a) $10 \times 10 \times 10$

    b) $\sqrt{60} \times \sqrt{60} \times 10$

    c) $2 \times 2 \times 150$

    d) $20 \times 20 \times 1.5$

15. Given that $f'(x) > 0$ on $(0, 2)$ and $f''(x) < 0$ on $(1, 2)$, which sketch best represents the graph of f?

16. Let $f(x) = \dfrac{x^2}{x^2 + 1}$. Which sketch best represents the graph of f?

17. A stone is tossed into a still pond, causing a circular ripple of water whose radius grows at a rate of $\frac{dr}{dt} = 10cm/sec$. Find the rate of change of the area of this circular ripple when the radius is $32cm$.

    a) 32π

    b) 64π

    c) 320π

    d) 640π

18. Which of the following is an x-coordinate of a point of inflection for the function $f(x) = 18\ln x + x^2 + 10$?

    a) 3

    b) 2

    c) 1

    d) 0

19. A bank pays 6% interest compounded monthly. How much will $\$1,000$ become after two years?

a) $2,000e^{0.06}$

b) $1,000e^{0.12}$

c) $1,000(1.06)^{24}$

d) $1,000(1.005)^{24}$

20. Simplify $\ln \dfrac{x^2\sqrt{x^2-1}}{e^x}$.

a) $\dfrac{\ln x^2 \ln \sqrt{x^2-1}}{\ln e^x}$

b) $\ln x^2 + \dfrac{1}{2}\ln x^2 - \dfrac{1}{2}\ln 1 - \ln e^x$

c) $2\ln x + \dfrac{1}{2}\ln(x-1) + \dfrac{1}{2}\ln(x+1) - x$

d) $e^{-x}\ln(x^2\sqrt{x^2-1})$

21. Find $\dfrac{d}{dx}(\ln(\ln x))$.

a) $\dfrac{1}{\ln x}$

b) $\dfrac{1}{x \ln x}$

c) $\dfrac{\ln x}{x}$

d) $\dfrac{x}{\ln x}$

22. Velocity of the car in m/sec at time t (in sec) is given by $v(t) = 1 + t^2$. Find the distance traveled by the car in the first three seconds.

a) 6

b) 9

c) 12

d) 15

23. A bank pays 5% interest compounded continuously. In how many years will the money double?

a) $15 \ln 2$

b) $20 \ln 2$

c) $25 \ln 2$

d) $30 \ln 2$

24. What is $\int \left(e^{-2x} + \frac{1}{x^2} + \frac{3}{x} \right) dx$?

a) $-\frac{e^{-2x}}{2} - \frac{1}{x} + 3 \ln |x| + c$

b) $\frac{e^{-2x}}{2} + \frac{1}{x} + 3 \ln |x| + c$

c) $-2e^{-2x} - 3\frac{1}{x} - \frac{3}{x^2} + c$

d) $e^{-2x} - \frac{3}{x^3} - \frac{3}{x^2} + c$

25. A car's velocity at time t is given by $v(t) = 3t^2$. Find the average velocity of the car during the time $1 \leq t \leq 3$.

a) 10

b) 11

c) 12

d) 13

26. If $f'(x) = \dfrac{2x}{x^2+1}$ and $f(0) = 0$, what is f?

a) $\ln(x^2+1)$

b) $\ln(x^2)$

c) $\ln(x^2)+1$

d) $\dfrac{x^2}{x^2+1}$

27. Find $\int_0^1 2xe^{x^2}dx$.

a) $e-1$

b) e

c) $2e-1$

d) $e-2$

28. Find $\int x\sqrt{x^2+4}\,dx$.

a) $\frac{1}{2}\sqrt[3]{(x^2+4)^2}+C$

b) $\frac{1}{2}x\sqrt{(x^2+4)^3}+C$

c) $\frac{1}{3}\sqrt{(x^2+4)^3}+C$

d) $2\sqrt[3]{(x^2+4)^2}+C$

29. Find $\dfrac{d}{dx}x^x$. Hint: use logarithmic differentiation.

a) x^x

b) $x \cdot x^x$

c) $x^x \ln x$

d) $x^x(1 + \ln x)$

30. Find the area enclosed by $y = 1 - x^2$ and the x-axis.

a) $\dfrac{1}{2}$

b) 1

c) $\dfrac{4}{3}$

d) $\dfrac{5}{3}$

31. KEY: 1c

a) 2d

b) 3c

c) 4a

d) 5b

e) 6b

f) 7b

g) 8b

h) 9d

i) 10d

j) 11d

k) 12a

l) 13b

m) 14a

n) 17d

o) 18a

p) 19d

q) 20c

r) 21b

s) 22c

t) 23b

u) 24a

v) 25d

w) 26a

x) 28c

y) 29d

z) 30c