Math 250	Quiz 2	Name: ____________
Section: 1	Summer 2009	Instructor: Vitaliy Gyrya
		Date: July 13, 2009

All work must be shown to receive full credit!

1. Find the general solution to the differential equation:

$$y'' - 3y' + 2y = 0.$$

Characteristic eq-n: $r^2 - 3r + 2 = 0 \iff (r-1)(r-2) = 0$

Roots $r_1 = 1$, $r_2 = 2$. Basic solutions: $\begin{cases} y_1(t) = e^t \\ y_2(t) = e^{2t} \end{cases}$

General solution: $\boxed{y(t) = c_1 e^t + c_2 e^{2t}.}$

2. Find the general solution to the differential equation:

$$y'' + y' + y = 0.$$

Characteristic eq-n: $r^2 + r + 1$

Roots $r_{1,2} = \frac{-1 \pm \sqrt{1-4}}{2} = \frac{-1}{2} \pm \sqrt{3}\, i/2$

General solution: $\boxed{y(t) = e^{-\frac{1}{2}t}\left(c_1 \cos\left(\frac{\sqrt{3}t}{2}\right) + c_2 \sin\left(\frac{\sqrt{3}t}{2}\right)\right).}$

3. Find the general solution to the differential equation:

$$y'' + 2y' + y = 0.$$

Characteristic eq-n: $r^2 + 2r + 1 = 0 \iff (r+1)^2 = 0$

Roots: $r_1 = -1 = r_2$

Basic solutions: $y_1(t) = e^{-t}$, $y_2(t) = te^{-t}$

General solution: $\boxed{y(t) = c_1 e^{-t} + c_2 t e^{-t}}$

4. Use the method of reduction of order to find a second solution of the differential equation:

$$t^2y'' + 3ty' - 8y = 0, \qquad t > 0,$$

Standard form

$$y'' + \underbrace{\frac{3}{t}}_{p(t)} y' + \underbrace{\frac{-8}{t^2}}_{q(t)} y = 0$$

knowing that $y_1(t) = t^2$ is a solution.

- Check that $y_1(t) = t^2$ is, indeed, a solution

$$t^2 \cdot 2 + 3t \cdot 2t - 8t^2 = (2+6-8)t^2 = 0 \checkmark.$$

- Look for a solution in the form $y_2(t) = v(t)\, y_1(t)$.

- Use the definition of the Wronskian

$$W(y_1, y_2)(t) = W(y_1, vy_1) = \det \begin{vmatrix} y_1 & vy_1 \\ y_1' & vy_1' + v'y_1 \end{vmatrix} = v' y_1^2 \qquad (1)$$

and Abel's Theorem

$$W(y_1, y_2) = c\, e^{-\int p(t)\,dt} = c\, e^{-\int \frac{3}{t}\,dt} = c\, e^{-3\ln t} = c\, t^{-3}. \qquad (2)$$

- Combine (1) and (2) to get an eq-n for v:

$$v' = \frac{1}{y_1^2} c t^{-3} = \frac{1}{(t^2)^2} c t^{-3} = c t^{-7} \Rightarrow v = \int c t^{-7} dt \underset{\uparrow}{=} t^{-6}$$

Constant c chosen appropriately

- Second solution is

$$\boxed{y_2(t)} = v(t)\, y_1(t) = t^{-6} t^2 = \boxed{t^{-4}}$$

y_2	y_2'	y_2''
t^{-4}	$-4t^{-5}$	$20t^{-6}$

- Check: $t^2 \cdot 20t^{-6} + 3t(-4t^{-5}) - 8t^{-4} = (20-12-8)t^{-4} = 0 \checkmark$

Answer: $\boxed{y_2(t) = t^{-4}}$