

All work must be shown to receive full credit!

1. A 1000m^3 room initially contains fresh air. At $t = 0$, a faulty heating system causes gas containing 10% carbon monoxide to be pumped into the room at a rate of 10m^3 per minute. A well-mixed air is vented out at the same rate.

(a) Introduce the variables and their meaning.

$W(t)$ - amount of air in the room.

$S(t)$ - amount of carbon monoxide

$\frac{S(t)}{W(t)}$ - concentration of carbon monoxide in the room.

(b) Write the differential equations, and give the initial conditions, that describe this event.

$$W'(t) = r_{in} - r_{out} = 10 - 10 = 0, \quad W(0) = 1000.$$

$$S'(t) = r_{in} C_{in} - r_{out} C_{out} = 10 \cdot 0.1 - 10 \frac{S(t)}{1000} = 1 - \frac{1}{100} S(t)$$

$$S(0) = 0.$$

(c) Solve the initial value problem.

$$w(t) \equiv 1000$$

$$S' + \frac{1}{100}S = 1 \quad | \cdot \mu$$

$$\mu' = \frac{1}{100}\mu \Rightarrow \mu(t) = e^{\frac{t}{100}}$$

$$\left(e^{\frac{t}{100}} S\right)' = e^{\frac{t}{100}}$$

$$e^{\frac{t}{100}} S = 100 e^{\frac{t}{100}} + C$$

$$S = 100 + C e^{-\frac{t}{100}}$$

From I.C.

$$S(0) = 0 \Rightarrow$$

$$C = -1000$$

$$S(t) = 100(1 - e^{-\frac{t}{100}})$$

2. Solve the following initial value problem

$$y' + y^2 = ty^2, \quad y(0) = 1.$$

$$y' = y^2(t-1)$$

$$\int \frac{y'}{y^2} dt = \int (t-1) dt$$

$$\frac{-1}{y} = \frac{1}{2}t^2 - t + C$$

$$y = \frac{-1}{\frac{1}{2}t^2 - t + C}$$

From I.C.

$$y(0) = 1 = \frac{-1}{0 - 0 + C} \Rightarrow$$

$$\Rightarrow C = -1 \Rightarrow$$

Solution

$$y(t) = \frac{-1}{\frac{1}{2}t^2 - t - 1}$$