$$\int e^{-t}\sin t\,dt = \int (-e^{-t})'\sin t\,dt \overset{\text{by parts}}{=}$$

$$= (-e^{-t})\sin t - \int (-e^{-t})\cos t\,dt =$$

$$= -e^{-t}\sin t - \int (e^{-t})'\cos t\,dt \overset{\text{by parts}}{=}$$

$$= -e^{-t}\sin t - \left[e^{-t}\cos t - \int e^{-t}(-\sin t)\,dt \right] =$$

$$= -e^{-t}(\sin t + \cos t) - \int e^{-t}\sin t\,dt .$$

LHS = RHS $\quad | + \int e^{-t}\sin t\,dt$

$$\Downarrow$$

$$2\int e^{-t}\sin t\,dt = -e^{-t}(\sin t + \cos t)$$

$$\Downarrow$$

$$\boxed{\int e^{-t}\sin t\,dt = -\frac{1}{2}e^{-t}(\sin t + \cos t)}$$