

Let $t_0 = 0$ - initial time

t_1 - time when object reached the highest point ($v(x(t_1)) = 0$)

Note the following trick:

$$t_1 = t_1 - t_0 = \int_{t_0}^{t_1} 1 dt = \left[\begin{array}{l} t = t(x) \\ dt = \frac{dt}{dx} dx \\ \text{Change bounds} \\ \text{of integration} \end{array} \right] = \int_{x(t_0)}^{x(t_1)} \left(\frac{dt}{dx} \right) dx$$

Note that $\frac{dx}{dt} = v \Rightarrow \boxed{\frac{dt}{dx} = \frac{1}{v}}!$

So,

$$t_1 = \int_{x(t_0)}^{x(t_1)} \frac{1}{v} dx = \int_0^{x_{\max}} \frac{1}{v} dx$$