

**PROBLEMS**

In each of Problems 1 through 8 find the general solution of the given differential equation.

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|-------------------------|-------------------------|
| 1. $y'' + 2y' - 3y = 0$ | 2. $y'' + 3y' + 2y = 0$ |
| 3. $6y'' - y' - y = 0$  | 4. $2y'' - 3y' + y = 0$ |
| 5. $y'' + 5y' = 0$      | 6. $4y'' - 9y = 0$      |
| 7. $y'' - 9y' + 9y = 0$ | 8. $y'' - 2y' - 2y = 0$ |

In each of Problems 9 through 16 find the solution of the given initial value problem. Sketch the graph of the solution and describe its behavior as  $t$  increases.

9.  $y'' + y' - 2y = 0$ ,  $y(0) = 1$ ,  $y'(0) = 1$
10.  $y'' + 4y' + 3y = 0$ ,  $y(0) = 2$ ,  $y'(0) = -1$
11.  $6y'' - 5y' + y = 0$ ,  $y(0) = 4$ ,  $y'(0) = 0$
12.  $y'' + 3y' = 0$ ,  $y(0) = -2$ ,  $y'(0) = 3$
13.  $y'' + 5y' + 3y = 0$ ,  $y(0) = 1$ ,  $y'(0) = 0$
14.  $2y'' + y' - 4y = 0$ ,  $y(0) = 0$ ,  $y'(0) = 1$
15.  $y'' + 8y' - 9y = 0$ ,  $y(1) = 1$ ,  $y'(1) = 0$
16.  $4y'' - y = 0$ ,  $y(-2) = 1$ ,  $y'(-2) = -1$
17. Find a differential equation whose general solution is  $y = c_1 e^{2t} + c_2 e^{-3t}$ .
18. Find a differential equation whose general solution is  $y = c_1 e^{-t/2} + c_2 e^{-2t}$ .
19. Find the solution of the initial value problem

$$y'' - y = 0, \quad y(0) = \frac{5}{4}, \quad y'(0) = -\frac{3}{4}.$$

Plot the solution for  $0 \leq t \leq 2$  and determine its minimum value.

20. Find the solution of the initial value problem

$$2y'' - 3y' + y = 0, \quad y(0) = 2, \quad y'(0) = \frac{1}{2}.$$

Then determine the maximum value of the solution and also find the point where the solution is zero.

21. Solve the initial value problem  $y'' - y' - 2y = 0$ ,  $y(0) = \alpha$ ,  $y'(0) = 2$ . Then find  $\alpha$  so that the solution approaches zero as  $t \rightarrow \infty$ .
22. Solve the initial value problem  $4y'' - y = 0$ ,  $y(0) = 2$ ,  $y'(0) = \beta$ . Then find  $\beta$  so that the solution approaches zero as  $t \rightarrow \infty$ .

In each of Problems 23 and 24 determine the values of  $\alpha$ , if any, for which all solutions tend to zero as  $t \rightarrow \infty$ ; also determine the values of  $\alpha$ , if any, for which all (nonzero) solutions become unbounded as  $t \rightarrow \infty$ .

23.  $y'' - (2\alpha - 1)y' + \alpha(\alpha - 1)y = 0$
24.  $y'' + (3 - \alpha)y' - 2(\alpha - 1)y = 0$

25. Consider the initial value problem

$$2y'' + 3y' - 2y = 0, \quad y(0) = 1, \quad y'(0) = -\beta,$$

where  $\beta > 0$ .

- (a) Solve the initial value problem.
- (b) Plot the solution when  $\beta = 1$ . Find the coordinates  $(t_0, y_0)$  of the minimum point of the solution in this case.
- (c) Find the smallest value of  $\beta$  for which the solution has no minimum point.