

Solution

Math 251
Section: 9

Quiz 2
Spring 2009

Name: _____
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All work must be shown to receive full credit!

1. Compute the following (indicate the identity you are using).

$$\begin{aligned}
 (a) \mathcal{L}\{u_1(t)(t-1)e^{2(t-1)}\sin(t-1)\}(s) & \stackrel{(13)}{=} e^{-s} \mathcal{L}\{t e^{2t} \sin(t)\}(s) \stackrel{(19)}{=} \\
 & \stackrel{(19)}{=} -e^{-s} \frac{d}{ds} \mathcal{L}\{e^{2t} \sin(t)\}(s) \stackrel{(14)}{=} -e^{-s} \frac{d}{ds} \mathcal{L}\{\sin(t)\}(s-2) \stackrel{(7)}{=} \\
 & \stackrel{(7)}{=} -e^{-s} \frac{d}{ds} \left(\frac{1}{(s-2)^2 - 1} \right) = e^{-s} \frac{2(s-2)}{((s-2)^2 - 1)^2}
 \end{aligned}$$

$$\begin{aligned}
 (b) \mathcal{L}^{-1}\left\{\frac{e^{-3s}}{s(s^2+1)}\right\}(t) & \stackrel{(13)}{=} u_3(t) \mathcal{L}^{-1}\left\{\frac{1}{s(s^2+1)}\right\}(t-3) \stackrel{(*)}{=} \\
 & = u_3(t) \mathcal{L}^{-1}\left\{\frac{1}{s} - \frac{s}{s^2+1}\right\}(t-3) \stackrel{(1), (5)}{=} \underline{u_3(t) [1 - \cos(t-3)]}
 \end{aligned}$$

$$\frac{1}{s(s^2+1)} = \frac{a}{s} + \frac{bs+c}{s^2+1} = \frac{(as^2+a) + (bs^2+cs)}{s(s^2+1)} = \frac{(a+b)s^2 + cs + a}{s(s^2+1)}$$

So,

$$\left\{ \begin{array}{l} a+b=0 \Rightarrow \boxed{b=-a=-1} \\ \hline c=0 \\ \hline a=1 \end{array} \right. \Rightarrow \underline{\frac{1}{s(s^2+1)} = \frac{1}{s} - \frac{s}{s^2+1}} \quad (*)$$

TABLE 6.2.1 Elementary Laplace Transforms

$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$	Notes
1. 1	$\frac{1}{s}, \quad s > 0$	Sec. 6.1; Ex. 4
2. e^{at}	$\frac{1}{s-a}, \quad s > a$	Sec. 6.1; Ex. 5
3. $t^n, \quad n = \text{positive integer}$	$\frac{n!}{s^{n+1}}, \quad s > 0$	Sec. 6.1; Prob. 27
4. $t^p, \quad p > -1$	$\frac{\Gamma(p+1)}{s^{p+1}}, \quad s > 0$	Sec. 6.1; Prob. 27
5. $\sin at$	$\frac{a}{s^2 + a^2}, \quad s > 0$	Sec. 6.1; Ex. 7
6. $\cos at$	$\frac{s}{s^2 + a^2}, \quad s > 0$	Sec. 6.1; Prob. 6
7. $\sinh at$	$\frac{a}{s^2 - a^2}, \quad s > a $	Sec. 6.1; Prob. 8
8. $\cosh at$	$\frac{s}{s^2 - a^2}, \quad s > a $	Sec. 6.1; Prob. 7
9. $e^{at} \sin bt$	$\frac{b}{(s-a)^2 + b^2}, \quad s > a$	Sec. 6.1; Prob. 13
10. $e^{at} \cos bt$	$\frac{s-a}{(s-a)^2 + b^2}, \quad s > a$	Sec. 6.1; Prob. 14
11. $t^n e^{at}, \quad n = \text{positive integer}$	$\frac{n!}{(s-a)^{n+1}}, \quad s > a$	Sec. 6.1; Prob. 18
12. $u_c(t)$	$\frac{e^{-cs}}{s}, \quad s > 0$	Sec. 6.3
13. $u_c(t)f(t-c)$	$e^{-cs}F(s)$	Sec. 6.3
14. $e^{ct}f(t)$	$F(s-c)$	Sec. 6.3
15. $f(ct)$	$\frac{1}{c}F\left(\frac{s}{c}\right), \quad c > 0$	Sec. 6.3; Prob. 25
16. $\int_0^t f(t-\tau)g(\tau) d\tau$	$F(s)G(s)$	Sec. 6.6
17. $\delta(t-c)$	e^{-cs}	Sec. 6.5
18. $f^{(n)}(t)$	$s^n F(s) - s^{n-1}f(0) - \dots - f^{(n-1)}(0)$	Sec. 6.2
19. $(-t)^n f(t)$	$F^{(n)}(s)$	Sec. 6.2; Prob. 28