

ADDITIONAL HOMEWORK PROBLEMS

1. The displacement $u(t, x)$ of a spring of length 10cm with ends clamped satisfies the differential equation

$$u_{tt} = 9u_{xx}.$$

Suppose the initial displacement of the string is

$$u(0, x) = \sin(10\pi x)$$

and the initial velocity of the string is

$$u_t(0, x) = 0.$$

Write down a formula for the displacement $u(t, x)$ of the string at any time $t > 0$. Simplify your answer.

2. The displacement $u(t, x)$ of a spring of length 10cm with ends clamped satisfies the differential equation

$$u_{tt} = 9u_{xx}.$$

Suppose the initial displacement of the string is

$$u(0, x) = 0$$

and the initial velocity of the string is

$$u_t(0, x) = \sin(10\pi x).$$

Write down a formula for the displacement $u(t, x)$ of the string at any time $t > 0$. Simplify your answer.

3. Find the solution of the Laplace equation

$$u_{xx} + u_{yy} = 0$$

on the rectangle

$$\{(x, y) \mid 0 < x < 5, \ 0 < y < 10\},$$

which satisfies the following Dirichlet boundary conditions

$$\begin{cases} u(x, 0) = 0 & \text{if } 0 < x < 5 \\ u(x, 10) = 0 & \text{if } 0 < x < 5 \\ u(0, y) = 0 & \text{if } 0 < y < 10 \\ u(5, y) = \sin(\pi y) & \text{if } 0 < y < 10 \end{cases}$$