Corrections

Suppose the solution to undamped oscillations is

$$u(t) = A\cos(\omega_0 t) + B\sin(\omega_0 t). \tag{1}$$

The standard form for the solution is

$$u(t) = R\cos(\omega_0 t - \delta). \tag{2}$$

To get the solution (1) into the form (2) perform the following steps:

Step 1: Find the amplitude

$$R = \sqrt{A^2 + B^2}.$$

Step 2: Multiply and divide (1) by R

$$\begin{aligned} u(t) &= A\cos(\omega_0 t) + B\sin(\omega_0 t) = \\ &= \sqrt{A^2+B^2}\left(\frac{A}{\sqrt{A^2+B^2}}\cos(\omega_0 t) + \frac{B}{\sqrt{A^2+B^2}}\sin(\omega_0 t)\right). \end{aligned}$$

Step 3: Find an angle δ such that

$$\begin{cases} \cos(\delta) = \frac{A}{\sqrt{A^2+B^2}} \\ \sin(\delta) = \frac{B}{\sqrt{A^2+B^2}}. \end{cases}$$

This will allow us to write

$$\begin{aligned} u(t) &= \sqrt{A^2+B^2}\left(\frac{A}{\sqrt{A^2+B^2}}\cos(\omega_0 t) + \frac{B}{\sqrt{A^2+B^2}}\sin(\omega_0 t)\right) = \\ &= R\Big(\cos(\delta)\cos(\omega_0 t) + \sin(\delta)\sin(\omega_0 t)\Big). \end{aligned}$$

Step 4: Use the trigonometric formula for cosine of a difference[1] to write

$$u(t) = R\Big(\cos(\delta)\cos(\omega_0 t) + \sin(\delta)\sin(\omega_0 t)\Big) = R\cos(\omega_0 t - \delta).$$

[1] Formula for cosine of a difference:

$$\cos(\alpha - \beta) = \cos(\alpha)\cos(\beta) - \sin(\alpha)\sin(\beta).$$