

Name	Equation	Solution
Heat	$\begin{cases} u_t = au_{xx}, \\ u(t, 0) = 0, \quad u(t, L) = 0, \\ u(0, x) = f(x). \end{cases}$	$u(t, x) = \sum_{n=1}^{\infty} c_n u_n(t, x)$ $u_n(t, x) = e^{-\left(\frac{n\pi}{L}\right)^2 at} \sin\left(\frac{n\pi x}{L}\right)$
Heat	$\begin{cases} u_t = au_{xx}, \\ u_x(t, 0) = 0, \quad u_x(t, L) = 0, \\ u(0, x) = f(x). \end{cases}$	$u(t, x) = \sum_{n=1}^{\infty} c_n u_n(t, x)$ $u_n(t, x) = e^{-\left(\frac{n\pi}{L}\right)^2 at} \cos\left(\frac{n\pi x}{L}\right)$
Wave	$\begin{cases} u_{tt} = a^2 u_{xx}, \\ u(t, 0) = 0, \quad u(t, L) = 0, \\ u(0, x) = f(x), \quad u_t(0, x) = g(x). \end{cases}$	$u(t, x) = \sum_{n=1}^{\infty} (c_n u_n(t, x) + d_n v_n(t, x))$ $u_n(t, x) = \sin\left(\frac{n\pi at}{L}\right) \sin\left(\frac{n\pi x}{L}\right)$ $v_n(t, x) = \cos\left(\frac{n\pi at}{L}\right) \sin\left(\frac{n\pi x}{L}\right)$
Laplace	$\begin{cases} u_{xx} + u_{yy} = 0, \\ u(x, 0) = 0, \quad u(0, y) = 0, \\ u(x, b) = 0, \quad u(a, y) = f(y). \end{cases}$	$u(x, y) = \sum_{n=1}^{\infty} c_n u_n(x, y)$ $u_n(x, y) = \sinh\left(\frac{n\pi x}{b}\right) \sin\left(\frac{n\pi y}{b}\right)$