Math 250 Quiz 1 Name: ______________________

Section: 4 Fall 2007 Instructor: Vitaliy Gyrya

Date: October 10, 2007

All work must be shown to receive full credit!

1. For each of the following differential equations **find the general solution**:

(a) $y'' + 4y' + y = 0$

$r^2 + 4r + 1 = 0 \qquad r_{1,2} = \frac{-4 \pm \sqrt{16-4}}{2} = -2 \pm \sqrt{3}$

$$\boxed{y(t) = c_1 e^{-2-\sqrt{3}} + c_2 e^{-2+\sqrt{3}}}$$

(b) $y'' + 2y' + y = 0$

$r^2 + 2r + 1 = 0 \quad , \qquad r_{1,2} = -1 \pm 0$

$$\boxed{y(t) = c_1 e^{-t} + c_2 t e^{-t}}$$

(c) $y'' + y = 0$

$r^2 + 1 = 0 \qquad r_{1,2} = \frac{0 \pm \sqrt{-4}}{2} = \pm i$

$$\boxed{y(t) = c_1 \cos(t) + c_2 \sin(t)}$$

2. One solution for the differential equation

$$x^2y'' + 3xy' + y = 0 \;; \qquad \underbrace{y'' + \frac{3}{x}y' + \frac{1}{x^2}y = 0}_{} \qquad (*)$$

is $y_1(x) = x^{-1}$.

Find the general solution for the above equation.

Look for a solution

$$y_2(x) = v(x)\, y_1(x)$$

Substitute y_2 into $(*)$ and recall that the terms with $v(x)$ cancel

$$v'' y_1 + v'\left[2y_1' + \frac{3}{x}y_1\right] = 0 \qquad \Big|\div y_1$$

$$v'' + v'\left[\frac{2y_1'}{y_1} + \frac{3}{x}\right] = 0$$

Recall that $y_1(x) = \frac{1}{x}$; $y_1'(x) = \frac{-1}{x^2}$

$$v'' + v'\left[\frac{-2x^{-2}}{x^{-1}} + 3x^{-1}\right] = 0$$

$$v'' + v'\cdot\left(\frac{1}{x}\right) = 0$$

$$v'' = \frac{-1}{x}v'$$

$$v' = e^{-\ln(x)} = x^{-1}; \quad v = \ln(x)$$

$$y_2(x) = v(x)y_1(x) = \ln(x)\, x^{-1}.$$

General solution: $y(x) = c_1 \frac{1}{x} + c_2 \frac{\ln(x)}{x}$