MATH 250
Final
May 11, 2007

Name:_____
Student Number:_____
Instructor:_____
Section:_____

There are **8** multiple choice questions and **6** partial credit questions. **In order to obtain full credit for the partial credit problems, all work must be shown. Credit will not be given for an answer not supported by work on a partial credit problem. THE USE OF CALCULATORS IS NOT PERMITTED IN THIS EXAMINATION.**

For multiple choice problems, write the letter of your choice in the space provided below.

Your Answer :	**Points awarded**
1. (5 pts) _____	Q. 9 (15 pts) _____
2. (5 pts) _____	Q. 10 (15 pts) _____
3. (5 pts) _____	Q. 11 (20 pts) _____
4. (5 pts) _____	Q. 12 (20 pts) _____
5. (5 pts) _____	Q. 13 (20 pts) _____
6. (5 pts) _____	Q. 14 (20 pts) _____
7. (5 pts) _____	
8. (5 pts) _____	

1. (5 points) Find two linearly *independent* vectors
   (a) $\begin{pmatrix}1\\1/2\end{pmatrix}, \begin{pmatrix}2\\1\end{pmatrix}$.
   (b) $\begin{pmatrix}1\\-1\end{pmatrix}, \begin{pmatrix}-2\\2\end{pmatrix}$.
   (c) $\begin{pmatrix}1\\1/2\end{pmatrix}, \begin{pmatrix}0\\0\end{pmatrix}$.
   (d) None of the above.

2. (5 points) Find two linearly *dependent* functions
   (a) t^2, $(t+1)^2$.
   (b) $(\sin t)^2$, $(\cos t)^2$.
   (c) $(\sin t)^2 + (\cos t)^2$, -2.
   (d) None of the above.

3. (5 points) The function $y_1(t) = t^2$ is a solution of $t^4y'' - t^3y' - ty' + 2y = 0$, $t > 0$. Find the general solution of this equation.
   (a) $y_3(t) = c_1t^2 + c_2t$
   (b) $y(t) = c_1t^2 + c_2t^2e^{-\frac{1}{2t^2}}$.
   (c) $y(t) = c_1e^t\cos(t) + c_2e^t\sin(t) + t^2$.
   (d) $y(t) = c_1e^{t/2}\cos\left(\frac{\sqrt{7}}{2}t\right) + c_2e^{t/2}\sin\left(\frac{\sqrt{7}}{2}t\right) + t^2$.

4. (5 points) Suppose $y_1(t)$ and $y_2(t)$ are solutions of $y'' + p(t)y' + q(t)y = g(t)$. Then one of the following is also a solution of $y'' + p(t)y' + q(t)y = g(t)$. Find it.
   (a) $y_3(t) = y_1(t) - y_2(t)$.
   (b) $y_3(t) = y_1(t) + y_2(t)$.
   (c) $y_3(t) = 2y_1(t) - y_2(t)$.
   (d) $y_3(t) = 2y_1(t) - 3y_2(t)$.

5. (5 points) Which of the following improper integrals defines the Laplace transform of

$$f(t) = \begin{cases} 0, & \text{if } 0 \le t < 1, \\ t-1, & \text{if } 1 \le t < 2, \\ 2t-3, & \text{if } 2 \le t. \end{cases} \quad ?$$

   (a) $\int_0^\infty (t-1)e^{-st}dt + \int_0^\infty (2t-3)e^{-st}dt.$
   (b) $\int_1^\infty (t-1)e^{-st}dt + \int_2^\infty (2t-3)e^{-st}dt.$
   (c) $\int_1^\infty (t-1)e^{-st}dt + \int_2^\infty (t-2)e^{-st}dt.$
   (d) $\int_0^\infty (3t-4)e^{-st}dt.$

6. (5 points) Suppose $y(t)$ is the solution of the initial value problem $dy/dx = y^2+1,\ y(0) = 0$. Find $y(\pi/4)$.
   (a) $4/3$.
   (b) $\sqrt{4/3}-1$
   (c) 1.
   (d) None of the above.

7. (5 points) The solution of the initial value problem

$$(x^2-4)\frac{dy}{dx} \;=\; xy-(x+1)^2, \qquad y(-1.5) \;=\; 1,$$

   is defined for
   (a) $x < -1$.
   (b) $-2 < x < -1$.
   (c) $|x| < 2$.
   (d) all real x.

8. (5 points) Find two distinct solutions for $y' = 2\sqrt{y},\ y(0) = 0,\ t \ge 0$.
   (a) $y_1 = 0,\ y_2 = u_1(t),\ u_1(t)$ is a Heaviside function.
   (b) $y_1 = t^2/4,\ y_2 = 0$.
   (c) $y_1 = t^2,\ y_2 = u_1(t)(t-1)^2,\ u_1(t)$ is a Heaviside function.
   (d) $y_1 = t^2/4,\ y_2 = t^2$.

9. (15 points) Consider the ordinary differential equation

$$y' = y^2 - 1.$$

(a) Find all its equilibrium solutions.

(b) Draw the phase line and determine stability of equilibrium solutions.

(c) Find all solutions to the differential equation.

10. (15 points) Find the general solution to the following linear equations

(a) $y'' - 4y' = 1$

(b) $y'' + 2y' + y = 1$

(c) $y'' + y' + y = 0$

11. (20 points)

(a) Solve the initial value problem

$$y'' + y' = \sin(\pi t)\delta(t - 1/2), y(0) = 1, y'(0) = 0.$$

(b) Write the solution $y(t)$ as a piecewise defined function (using curly braces instead of Heaviside functions).

12. (20 points)

(a) Find eigenvalues and eigenvectors of

$$A = \begin{pmatrix} -3 & 2 \\ 2 & -3 \end{pmatrix}$$

(b) Find the general solution to the system $X'(t) = AX$, where A is given in part a), and X is a two-dimensional vector.

(c) Describe the behavior of solutions of the system in part b) as t approaches positive infinity.

(d) Solve the initial value problem for the system in part b) when

$$X = \begin{pmatrix} 2 \\ 0 \end{pmatrix}.$$

13. (20 points) Find eigenvalues and classify the type of critical point at the origin of the linear system $X'(t) = AX$ if the 2×2 matrix A is given as

Matrix A	Eigenvalues	Type of Critical Point	Stability

(a) $\begin{pmatrix} -1 & 0 \\ 0 & 2 \end{pmatrix}$

(b) $\begin{pmatrix} -2 & 1 \\ -1 & -2 \end{pmatrix}$

(c) $\begin{pmatrix} -1 & 0 \\ 0 & -2 \end{pmatrix}$

(d) $\begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$

(e) $\begin{pmatrix} 2 & 1 \\ -1 & 2 \end{pmatrix}$

(f) $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

(g) For the first two cases ((a) and (b)) above draw the phase portraits.

14. (20 points)

(a) Find all equilibrium points of $dx/dt = x(y-1)$, $dy/dt = (x-y)$

(b) Determine stability of these equilibrium points.

Laplace Transform table

$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$
1. 1	$\frac{1}{s}, \quad s > a$
2. e^{at}	$\frac{1}{s-a}, \quad s > a$
3. te^{at}	$\frac{1}{(s-a)^2}, \quad s > a$
4. t^2e^{at}	$\frac{2}{(s-a)^3}, \quad s > a$
5. t^ne^{at}	$\frac{n!}{(s-a)^{n+1}}, \quad s > a$
6. $e^{at}\sin bt$	$\frac{b}{(s-a)^2+b^2}, \quad s > a$
7. $e^{at}\cos bt$	$\frac{s-a}{(s-a)^2+b^2}, \quad s > a$
8. $u_c(t)f(t-c)$	$e^{-cs}F(s)$
9. $\delta(t-c)$	e^{-cs}
10. $f(t)\delta(t-c)$	$e^{-cs}f(c)$
11. $f'(t)$	$sF(s) - f(0)$
12. $f''(t)$	$s^2F(s) - sf(0) - f'(0)$