

NAME : _____

MATH 250
Final Exam
December 15, 2003

ID : _____

INSTRUCTOR : _____

There are **11** questions on this exam. Please read each problem carefully before you begin. All work must be shown - **credit will not be given for an answer unsupported by work.**

NO CALCULATORS ARE ALLOWED.

1: _____
2: _____
3: _____
4: _____
5: _____
6: _____
7: _____
8: _____
9: _____
10: _____
11: _____
Total: _____

1. (9 points) Sketch the integral curve of the equation

$$\frac{dy}{dt} = y(4 - y)$$

that passes through the point $(0, 1)$.

2. (9 points) Solve the initial value problem

$$\begin{aligned}\frac{dy}{dt} &= \frac{2y}{t} \\ y(1) &= 1.\end{aligned}$$

3. (9 points) Solve the initial value problem

$$\begin{aligned}\frac{dy}{dt} + ty &= t^{5/2} \\ y(1) &= 1.\end{aligned}$$

4. (9 points) Determine the Wronskian of the two solutions $y_1(t)$ and $y_2(t)$ of the equation

$$y'' = ty,$$

if it is known that $y_1(0) = 1$, $y_2(0) = 0$ and $y_2'(0) = 3$.

5. (9 points) Find the general solution of the equation

$$y'' = \frac{2y}{t^2},$$

given that $y_1(t) = t^2$ is a solution.

6. (9 points) Find the general solution of

$$y'' - y = e^{3t} + e^{-t}.$$

7. (9 points) Find the general solution of the equation

$$y''' + 8y = 0$$

8. (9 points) Find the general solution of $\vec{x}' = A\vec{x}$ if

$$A = \begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}.$$

9. (9 points) Find the general solution of $\vec{x}' = A\vec{x}$ if

$$A = \begin{bmatrix} 3 & 2 & 0 \\ 2 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix}.$$

10. (9 points) Find the general solution of $\vec{x}' = A\vec{x} + \vec{P}$ if

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \text{ and } \vec{P} = \begin{bmatrix} e^t \\ e^{-t} \end{bmatrix}.$$

11. (9 points) Let

$$\vec{y}_1(t) = \begin{bmatrix} t \\ \sin(t) \\ \cos(t) \end{bmatrix}, \vec{y}_2(t) = \begin{bmatrix} 0 \\ t^2 + 1 \\ e^t \end{bmatrix} \text{ and } \vec{y}_3(t) = \begin{bmatrix} 0 \\ 0 \\ e^t \end{bmatrix}.$$

Let $A(t)$ be a 3×3 matrix such that its entries are continuous functions on $(-\infty, \infty)$. Find the Wronskian of \vec{y}_1, \vec{y}_2 and \vec{y}_3 . Explain why these three vector functions can not all be solutions of the system $\vec{y}' = A\vec{y}$.