MATH 250
Second Midterm Exam
Mar 29, 2006

Name:_____
Student Number:_____
Instructor:_____
Section:_____

There are **6** multiple choice questions and **5** partial credit questions. **In order to obtain full credit for the partial credit problems, all work must be shown. Credit will not be given for an answer not supported by work on a partial credit problem. THE USE OF CALCULATORS IS NOT PERMITTED IN THIS EXAMINATION.**

For multiple choice problems, write the letter of your choice in the space provided below.

Your Answer :

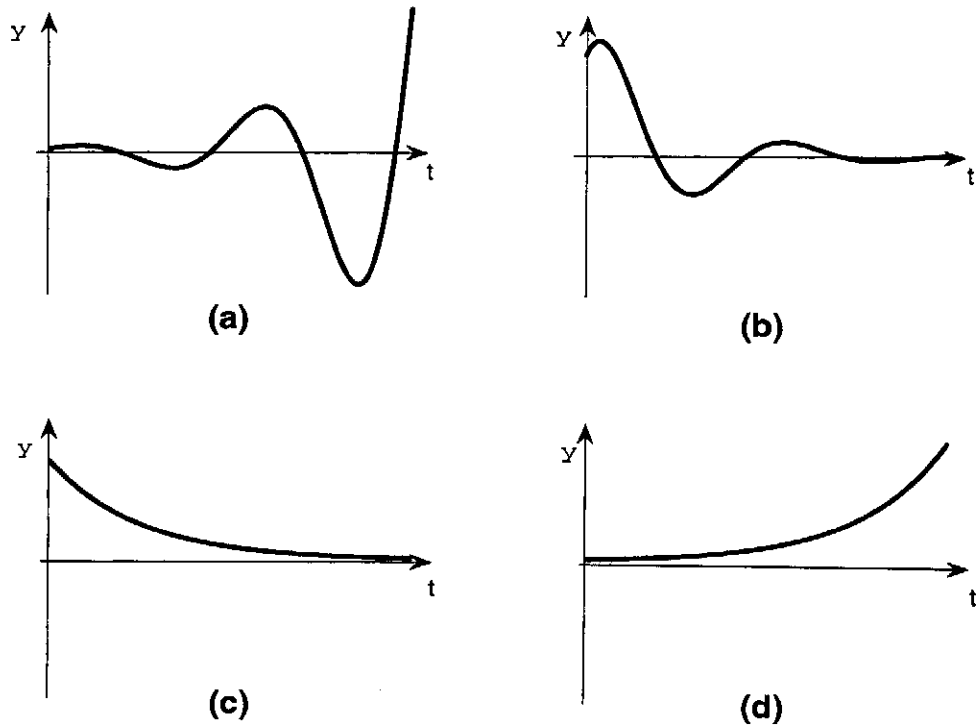
1. (5 pts) _____
2. (5 pts) _____
3. (5 pts) _____
4. (5 pts) _____
5. (5 pts) _____
6. (5 pts) _____

Points awarded

Q. 7 (10 pts) _____
Q. 8 (20 pts) _____
Q. 9 (10 pts) _____
Q. 10 (15 pts) _____
Q. 11 (15 pts) _____
total _____

1. (5 points) Which of the following figures represents a solution of the differential equation

$$8y'' - 2y' + \frac{5}{4}y = 0?$$



2. (5 points) Consider a spring-mass system with the displacement of the mass given by

$$u(t) = 3\cos(2t) + 4\sin(2t),$$

which of the following statement is **not** true?

(a) The initial displacement at $t = 0$ is 3 and the initial velocity is 8 .

(b) The motion of the mass is periodic with period π.

(c) The system is oscillating with a phase angle $\delta = \arctan\frac{3}{4}$.

(d) The system is oscillating with constant amplitude equal to 5.

3. (5 points) The Laplace transform of $f(t) = e^{2t}\cos(t)$ is,

   (a) $-\frac{s-2}{s^2-4s+3}$

   (b) $\frac{s-2}{s^2-4s+5}$

   (c) $-\frac{s-2}{s^2-4s+5}$

   (d) $\frac{s-2}{s^2-4s+3}$

4. (5 points) Which of the following is the solution of the IVP

$$y'' + 8y' + 16y = 0, \quad y(0) = 1, \quad y'(0) = 0?$$

   (a) $y(t) = e^{-4t}$

   (b) $y(t) = te^{-4t}$

   (c) $y(t) = e^{-4t} + 4te^{-4t}$

   (d) $y(t) = e^{4t} - 4te^{4t}$

5. (5 points) Using the method of undetermined coefficients, a **particular** solution of the differential equation,

$$y'' + y = 2\cos(t) + \sin(2t)$$

has the form:

(a) $A\cos(t) + B\sin(2t)$

(b) $A\cos(t) + B\sin(t) + C\cos(2t) + D\sin(2t)$

(c) $t\big(A\cos(t) + B\sin(t)\big) + C\cos(2t) + D\sin(2t)$

(d) $t\big(A\cos(t) + B\sin(t)\big) + t\big(C\cos(2t) + D\sin(2t)\big)$

6. (5 points) Using the method of undetermined coefficients, the **general** solution of the differential equation,

$$y'' + 5y' + 6y = t^2 + 1$$

has the form:

(a) $At^2 + Bt + C.$

(b) $c_1e^{-2t} + c_2e^{-3t} + At^2 + Bt + C.$

(c) $c_1e^{2t} + c_2e^{3t} + At^2 + Bt + C.$

(d) $c_1e^{-2t} + c_2e^{-3t}.$

7. (10 points) Solve the initial value problem,

$$y'' + 25y = 10\sin(5t), \quad y(0) = 1, \quad y'(0) = 0.$$

8. (20 points) A spring is streched 0.2 m by a force of 2 newtons. A mass of 2 kg is hung from the spring and is also attached to a viscous damper that exerts a force of 20 newtons when the velocity of the mass is 5m/sec. The mass is pulled down 0.05 m below its equilibrium position and given an initial upward velocity of 0.15 m/sec.

   (a) Set up an initial value problem to describe the motion of the mass.

   (b) Determine the position of the mass as a function of time.

   (c) Determine the quasi-period of the system and the time at which the mass first passes through its equilibrium position.

9. (10 points) Use the method of reduction of order to find the general solution of the differential equation:

$$t^2y'' - t(t+2)y' + (t+2)y = 0, \quad t > 0,$$

knowing that $y_1(t) = t$ is a solution. (NO credit will be given to unjustified answers.)

11. (15 points) Compute the inverse Laplace transform of the function,

$$F(s) = \frac{2s - 1}{(s + 1)(s^2 + 7s + 12)}.$$

TABLE 6.2.1 Elementary Laplace Transforms

$f(t) = \mathscr{L}^{-1}\{F(s)\}$	$F(s) = \mathscr{L}\{f(t)\}$	Notes
1. 1	$\frac{1}{s}, \quad s > 0$	Sec. 6.1; Ex. 4
2. e^{at}	$\frac{1}{s-a}, \quad s > a$	Sec. 6.1; Ex. 5
3. t^n; n = positive integer	$\frac{n!}{s^{n+1}}, \quad s > 0$	Sec. 6.1; Prob. 27
4. t^p, $p > -1$	$\frac{\Gamma(p+1)}{s^{p+1}}, \quad s > 0$	Sec. 6.1; Prob. 27
5. $\sin at$	$\frac{a}{s^2+a^2}, \quad s > 0$	Sec. 6.1; Ex. 6
6. $\cos at$	$\frac{s}{s^2+a^2}, \quad s > 0$	Sec. 6.1; Prob. 6
7. $\sinh at$	$\frac{a}{s^2-a^2}, \quad s > \lvert a\rvert$	Sec. 6.1; Prob. 8
8. $\cosh at$	$\frac{s}{s^2-a^2}, \quad s > \lvert a\rvert$	Sec. 6.1; Prob. 7
9. $e^{at}\sin bt$	$\frac{b}{(s-a)^2+b^2}, \quad s > a$	Sec. 6.1; Prob. 13
10. $e^{at}\cos bt$	$\frac{s-a}{(s-a)^2+b^2}, \quad s > a$	Sec. 6.1; Prob. 14
11. $t^n e^{at}$, n = positive integer	$\frac{n!}{(s-a)^{n+1}}, \quad s > a$	Sec. 6.1; Prob. 18
12. $u_c(t)$	$\frac{e^{-cs}}{s}, \quad s > 0$	Sec. 6.3
13. $u_c(t)f(t-c)$	$e^{-cs}F(s)$	Sec. 6.3
14. $e^{ct}f(t)$	$F(s-c)$	Sec. 6.3
15. $f(ct)$	$\frac{1}{c}F\left(\frac{s}{c}\right), \quad c > 0$	Sec. 6.3; Prob. 19
16. $\int_0^t f(t-\tau)g(\tau)\,d\tau$	$F(s)G(s)$	Sec. 6.6
17. $\delta(t-c)$	e^{-cs}	Sec. 6.5
18. $f^{(n)}(t)$	$s^nF(s) - s^{n-1}f(0) - \cdots - f^{(n-1)}(0)$	Sec. 6.2
19. $(-t)^n f(t)$	$F^{(n)}(s)$	Sec. 6.2; Prob. 28