Name: ____________

MATH 250 Student Number: ____________

Midterm Exam II Instructor: ____________

November 07, 2005 Section: ____________

This exam has 12 questions for a total of 100 points. There are **4** partial credit questions. **In order to obtain full credit for partial credit problems, all work must be shown. Credit will not be given for an answer not supported by work.**

THE USE OF CALCULATORS or ANY OTHER ELECTRONIC DEVICES IS NOT PERMITTED IN THIS EXAMINATION.
At the end of the examination, the booklet will be collected.

Do not write in this box.

1: ____
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Total: ____

1. (5 points) Which of the following pairs of functions is linearly dependent?

   (a) $f(t) = \cos 2t$, $g(t) = \sin 2t$;

   (b) $f(t) = e^{2t}$, $g(t) = e^{-2t}$;

   (c) $f(t) = \cos t$, $g(t) = \sin\left(t - \frac{\pi}{2}\right)$;

   (d) $f(t) = t^2$, $g(t) = 2t$.

2. (5 points) Suppose y_1 and y_2 are two solutions of the second order linear differential equation

$$t\,y'' + 3\,y' + (2\sin t)\,y = 0.$$

   What is their Wronskian $W(y_1, y_2)(t)$ as a function of time if $W(y_1, y_2)(1) = 2$?

   (a) $W(y_1, y_2)(t) = 2t^3$;

   (b) $W(y_1, y_2)(t) = 2e^{-t^3}$;

   (c) $W(y_1, y_2)(t) = \dfrac{2}{t^3}$;

   (d) $W(y_1, y_2)(t) = 2e^{t^3-1}$.

3. (5 points) Consider the following initial value problem describing a mechanical system:

$$2y'' + 8y = 0, \quad y(0) = 0 \text{ m}, \quad y'(0) = 6 \text{ m/s}.$$

What is the amplitude of the resulting oscillation?

(a) 2 meters;

(b) 3 meters;

(c) 4 meters;

(d) 6 meters.

4. (5 points) Let $y(t)$ be a solution to the differential equation

$$y'' + 4y' + 4y = 0.$$

Which one of the following statements concerning $\lim_{t\to\infty} y(t)$ is **true**?

(a) Sometimes $y(t) \to 0$ and sometimes $y(t) \to \infty$ depending on the initial conditions $y(t)$ satisfies.

(b) For any solution $y(t)$, the limit is always zero.

(c) The limit does not exist because $y(t)$ oscillates.

(d) The limit is never zero for any solution $y(t)$.

5. (5 points) Which one of the following describes the general solution to the differential equation

$$9y'' - 12y' + 4y = 2e^{\frac{2}{3}t} \ ?$$

C_1, C_2 and A below are constants.

(a) $y = C_1 e^{\frac{2}{3}t} + C_2 t e^{\frac{2}{3}t} + A e^{\frac{2}{3}t}$;

(b) $y = C_1 e^{\frac{2}{3}t} + C_2 t e^{\frac{2}{3}t} + A t e^{\frac{2}{3}t}$;

(c) $y = C_1 e^{\frac{2}{3}t} + C_2 t e^{\frac{2}{3}t} + A t^2 e^{\frac{2}{3}t}$;

(d) $y = C_1 e^{\frac{2}{3}t} + C_2 e^{\frac{2}{3}t} + A t e^{\frac{2}{3}t}$.

6. (5 points) Consider the system of forced vibrations $27y'' + ky = 4\cos(t/3)$. For what value of k will the system undergo resonance?

(a) 3;

(b) $3\sqrt{3}$;

(c) 4;

(d) 9.

7. (5 points) Find a suitable form of a particular solution of

$$y'' + y = 2e^{-t} + \cos t + 2t.$$

A_1, A_2, A_3, A_4, and A_5 below are constants.

(a) $Y(t) = A_1 e^{-t} + A_2 \cos t + A_3 t$,

(b) $Y(t) = A_1 e^{-t} + A_2 \sin t + A_3 \cos t + A_4 t + A_5$,

(c) $Y(t) = A_1 e^{-t} + A_2 t \sin t + A_3 t \cos t + A_4 t + A_5$,

(d) $Y(t) = A_1 t e^{-t} + A_2 t \sin t + A_3 t \cos t + A_4 t$.

8. (5 points) Consider the differential equation

$$4y'' + 4y' + cy = 0.$$

Find the value of c so that the function

$$y(t) = C_1 e^{-\frac{t}{2}} + C_2 t e^{-\frac{t}{2}},$$

where C_1 and C_2 are constants, is a solution of this equation.

(a) $c = 2$;

(b) $c = 1$;

(c) $c = \frac{1}{2}$;

(d) $c = -\frac{1}{4}$.

9. (15 points) Consider the following differential equation

$$ty'' + (1 - 2t)y' + (t - 1)y = 0, \quad t > 0.$$

(a) (2 points) Verify that $y_1(t) = e^t$ is a solution to the given equation.

(b) (10 points) Find a second linearly independent solution $y_2(t)$ to the given equation. **Explain** why the given function $y_1(t)$ and obtained function $y_2(t)$ are **linearly independent**.

(c) (3 points) What is the general solution of the given equation?

10. (15 points) Use the method of undetermined coefficients to find the general solution of the following differential equation

$$y'' + y' - 2y = 2te^t.$$

11. (20 points) A mass of $m = 2\ kg$ is suspended by a spring with spring constant $k\ N/m$. The system is immersed in a fluid with damping constant $\gamma\ kg/sec$. In absence of external force, the differential equation for this system is

$$mu'' + \gamma u' + ku = 0,$$

where $u(t)$ is the displacement of the mass from the equilibrium position at time t. Assume that g, the acceleration due to gravity, is $10\ m/sec^2$.

(a) (3 points) Suppose the mass of $2\ kg$ stretches the spring $40\ cm$. What is k?

(b) (3 points) For what value(s) of γ will the system be critically damped?

(c) (10 points) Determine the position of the mass if the mass is released from the equilibrium position with an upward velocity of $1\ m/sec$ and the damping constant $\gamma = 16\ kg/sec$.

(d) (4 points) Find the quasi-period and quasi-frequency of this motion.

12. (10 points) Use the definition to find the Laplace transform of the function

$$f(t) = t - 2.$$