

Math 230

Quiz 5

Name: _____

Section: 1 & 2

Fall 2006

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All work must be shown to receive full credit!

1. (3 points) Is the vector field \mathbf{F} conservative?

$$\mathbf{F}(x, y) = \begin{bmatrix} \cos(xy) - xy \sin(xy) \\ -x^2 \sin(xy) \end{bmatrix}$$

Solution:

$$\mathbf{F} = \begin{bmatrix} P \\ Q \end{bmatrix} \stackrel{?}{=} \begin{bmatrix} f_x \\ f_y \end{bmatrix}.$$

The test is to check if $(f_x)_y \stackrel{?}{=} (f_y)_x$, i.e. $P_y \stackrel{?}{=} Q_x$.

$$\begin{aligned} P_y &= -x \sin(xy) - x \sin(xy) - x^2 y \cos(xy), \\ Q_y &= -2x \sin(xy) - x^2 y \cos(xy). \end{aligned}$$

We obtained $P_y = Q_x$, hence $\mathbf{F}(x, y)$ is a conservative vector field.

The potential function for $\mathbf{F}(x, y)$ is

$$f(x, y) = x \cos(xy).$$

(3 points) Compute the line integral

$$\int_C \mathbf{F} \cdot d\mathbf{r}, \quad \text{where } C : x^2 + y^2 = 4, \quad y \geq 0.$$

Assume that the curve C goes from $(2, 0)$ to $(-2, 0)$.

Solution:

Using the fundamental theorem of line integrals

$$\begin{aligned} \int_C \mathbf{F} \cdot d\mathbf{r} &= f(-2, 0) - f(2, 0) = (-2) \cos(-2 \cdot 0) - 2 \cos(2 \cdot 0) = \\ &= -2 - 2 = -4. \end{aligned}$$

2. (4 points) Compute the line integral

$$\int_C xy \, ds$$

where C is a line segment from $(0, 0)$ to $(1, 1)$.

Solution:

Choose the parametrization for the line segment:

$$\begin{cases} x(t) = t, \\ y(t) = t, \end{cases} \quad 0 \leq t \leq 1.$$

Then

$$\int_C xy \, ds = \int_0^1 x(t)y(t)\sqrt{(x'(t))^2 + (y'(t))^2} \, dt = \int_0^1 t^2\sqrt{2} \, dt = \frac{\sqrt{2}}{3}.$$

