Formulas Sheet

Curvature:

$$\kappa = \frac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3}.$$

Components of acceleration:

$$a_{T} = v' = \frac{\mathbf{r}'(t) \cdot \mathbf{r}''(t)}{|\mathbf{r}'(t)|}, \qquad a_{N} = \kappa v^2 = \frac{|\mathbf{r}'(t) \times \mathbf{r}''(t)|}{|\mathbf{r}'(t)|}.$$

Directional derivative:

$$D_{\vec{a}} f(x_0, y_0) = \vec{a} \cdot \nabla f(x_0, y_0), \qquad \text{for } |\vec{a}| = 1.$$

Length of a curve $\mathbf{r}(t)$, $a \le t \le b$:

$$L = \int_a^b |\mathbf{r}'(t)| \; dt.$$

Integral in polar coordinates:

$$\iint_R f(x,y) \; dA = \int_\alpha^\beta \int_a^b f(r\cos\theta, r\sin\theta) \, r \; dr \, d\theta.$$

Integral in cylindrical coordinates:

$$\iiint_E f(x,y,z) \; dV = \int_\alpha^\beta \int_{h_1(\theta)}^{h_2(\theta)} \int_{u_1(r,\theta)}^{u_2(r,\theta)} f(r\cos\theta, r\sin\theta, z) \, r \; dz \, dr \, d\theta.$$

Integral in spherical coordinates:

$$\iiint_E f(x,y,z) \, dV = \int_c^d \int_\alpha^\beta \int_a^b f(\rho\sin\phi\cos\theta, \rho\sin\phi\sin\theta, \rho\cos\phi) \, \rho^2 \sin\phi \, d\rho \, d\theta \, d\phi.$$

Area of a surface $z = f(x,y)$ above the region R:

$$A = \iint_R \sqrt{[f_x(x,y)]^2 + [f_y(x,y)]^2 + 1} \; dA.$$

Change of coordinates in integration:

$$\iint_R f(x,y) \; dx \, dy = \iint_S f(x(u,v), y(u,v)) \left| \frac{\partial(x,y)}{\partial(u,v)} \right| \; du \, dv.$$

Jacobian:

$$\left| \frac{\partial(x,y)}{\partial(u,v)} \right| = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial v} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial v} \end{vmatrix}.$$

Fundamental Theorem of line integrals:

$$\int_C \nabla f \cdot \, d\mathbf{r} = f(\mathbf{r}(b)) - f(\mathbf{r}(a)).$$

Green's Theorem (compare with Stokes Theorem):

$$\iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) \, dA = \int_C P\,dx \; + \; Q\,dy = \int_C \begin{bmatrix} P \\ Q \end{bmatrix} \cdot d\mathbf{r}.$$

Stoke's Theorem:

$$\iint_S \operatorname{curl} \mathbf{F} \cdot \mathbf{n} \; dS = \int_C \mathbf{F} \cdot \, d\mathbf{r}, \qquad \mathbf{n} \text{ – is the normal to the surface.}$$

Divergence Theorem:

$$\iiint_E \operatorname{div} \mathbf{F} \; dV = \iint_S \mathbf{F} \cdot \mathbf{n} \; dS, \qquad \mathbf{n} \text{ – is the normal to the surface.}$$