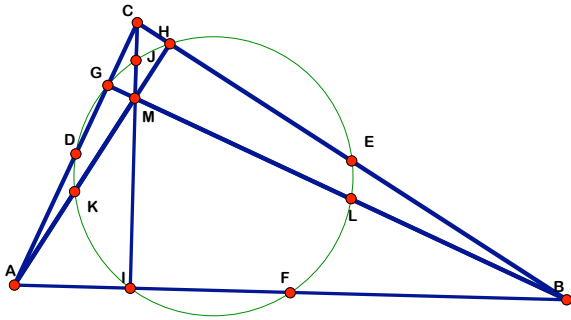


Homework 8
Due Friday, April 4.

1. Given any triangle ABC, let M be the point of intersection of the altitudes. Prove that the following nine points lie on the same circle:

- midpoints D,E,F of the three sides;
- points G,H,I, where altitudes meet the sides;
- midpoints J,K,L, of the segments MA,MB,MC.



Hint: Prove that KDEL is a rectangle. Prove that KDEL are equidistant from the point O the intersection of the diagonals DL and KE. It means these four point lie on the same circle centered at O and diameter $IDL=KEI$.

Prove that the point H also lie on this circle (use the fact that $\angle KHE=90^\circ$).

Prove that KJEF is a rectangle. Prove that the point of intersection of the diagonals of this rectangle is the same point O as before and points J,F lie on the same circle as before. Prove that G is on this circle.

Prove that DJLF is a rectangle and the point of intersection of diagonals is again O. Prove that I is on this circle.

2. Prove, using Ceva's theorem that angle bisectors intersect at one point.

3. Prove, using Ceva's theorem that altitudes intersect at one point.

4. In a $\triangle ABC$ a point D divides the side AC in the ratio 1:2 (AD is smaller) and a point E divides the side BC in the ratio 1:3 (BE is smaller). Let F is the point of intersection of AE and BD, and G is the point of intersection of the lines CF and AB. Find in what ratio G divides AB.

5. Let ABC is a triangle, points D and E are on the sides AC and BC such that $DE \parallel AB$. Let F is the point of intersection of AE and BD, and G is the point of intersection of the lines CF and AB. Find in what ratio G divides AB.