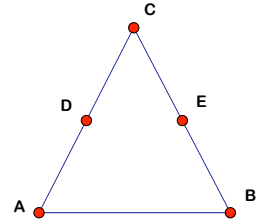


## Homework 4, Solutions.

Problem 16.  $\triangle ABC$  is isosceles triangle,  $|AC|=|BC|$ . D and E are the midpoints of the sides AC and BC. Prove that  $DE \parallel AB$ .



Proof:

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| 1. $\triangle ABC$ is isosceles.                         | 1. Given.  |
| 2. $\angle CAB = \angle CBA$ .                           | 2. Pons Asinorum (Theorem 13).                                   |
| 3. $ AC  =  BC $ .                                       | 3. Given.  |
| 4. $ DC  = \frac{1}{2} AC $ , $ EC  = \frac{1}{2} BC $ . | 4. Given.  |
| 5. $ DC  =  EC $ .                                       | 5. From 3. and 4.  |
| 6. $\triangle DEC$ is isosceles.                         | 6. From 5.   |
| 7. $\angle CDE = \angle CED$ .                           | 7. Pons Asinorum (Theorem 13).                                   |
| 8. $\angle CAB = \frac{180^\circ - \angle C}{2}$         | 8. Theorem 20 (the sum of angles of a triangle is $180^\circ$ )  |
| 9. $\angle CDE = \frac{180^\circ - \angle C}{2}$         | 9. Theorem 20 (the sum of angles of a triangle is $180^\circ$ )  |
| 10. $DE \parallel AB$                                    | 10. Theorem 18 (corresponding angles =, then lines $\parallel$ ) |

Problem 25. Assume lines  $a$  and  $b$  are parallel and a line  $c$  intersects  $a$ . Prove that  $c$  intersects  $b$ .

Proof (by contradiction):

Assume that  $c$  does not intersect  $b$ .

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|--|-----------------------|
| 1. $c \parallel b$ .   | 1. Assumption.        |
| 2. $a \parallel b$ .   | 2. Given.             |
| Denote the point of intersection of $c$ and $a$ as D.                                      |                       |
| 3. D is on $a$ and $c$ .   | 3. Given.             |
| 4. There are two lines $a$ and $c$ through the point D that are parallel to the line $b$ . | 4. From 1., 2. and 3. |
| 5. There is only one line through the point D that is parallel to the line $b$ .           | 5. Axiom 5.1          |

4. and 5. contradict each other, hence our assumption was wrong. It means line  $c$  intersects line  $b$ .