Homework 3. Solutions.

1. Prove that if in $\triangle ABC$ $\angle A=\angle B$, then $|AC|=|BC|$.

Proof:	Reasons:
1. $\angle A=\angle B$.	1. Given.
2. $|AB|=|BA|$.	2. Properties of the distance.
3. $\triangle ABC\cong\triangle BAC$.	3.ASA.
4. $|AC|=|BC|$. Hence, by deifinition $\triangle ABC$ is isosceles.	4. Converse of the theorem 11: in congruent triangles corresponding sides are equal.

5. Prove that the locus of points equidistant from points A and B is the segment bisector of the segment AB.

We need to prove that

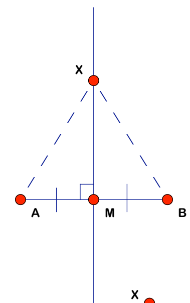
1) If a point X belongs to the segment bisector of the segment AB, then X is equidistant from points A and B.

2) If a point X equidistant from points A and B, then X belongs to the segment bisector of the segment AB.

If we prove these two statements then we solve the problem.

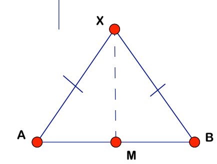
Proof of 1):

1. $|AM|=|MB|$.	1. Given.
2. $\angle AMX=\angle BMX$.	2. Given.
3. $\triangle AMX\cong\triangle BMX$.	3. SAS.
4. $|AX|=|BX|$. So X is equidistant from A and B.	4. Converse of the Th. 11 (corresponding sides of congruent triangles are equal).



Proof of 2):

1. $|AX|=|BX|$.	1. Given.
2. $\triangle ABX$ is isosceles.	2. From 1.
3. If M is midpoint, then XM is the altitude.	3. Problem 1 from this homework.
4. $XM\perp AB$	4.From 3.
5. XM is the perpendicular segment bisector.	5. From the definition of the perp. segment bisector.



7. Prove that if $\triangle ABC\cong\triangle A'B'C'$, then the lengths of corresponding medians are equal.

We have two different ways to prove that two segments are equal .
1. Show that there exists an isometry that maps first segment on the second.
2. Show that they are corresponding sides of two congruent triangles.
We solve problem 7 using first approach and 8 using second. Both problems could be solved easily using both approaches.

We know that $\triangle ABC\cong\triangle A'B'C'$, so we know that there exists an isometry f such that $f(\triangle ABC)=\triangle A'B'C'$. If we prove that $f(M)=M'$, where M and M' are midpoints of AB and A'B', then we are done. The median AM will be mapped onto the median A'M', but isomery preserves length. So the main part is to show that $f(M)=M'$.

Proof:

1. $|AM|=|MB|$.	1. Given.
2. $|f(A)f(M)|=|f(M)f(B)|$ or $|A'f(M)|=|f(M)B'|$.	2. Isometry preserves distances.
3. $f(M)$ is the midpoint of the segment $|A'B'|$.	3. From 2.
4. $M'=f(M)$.	4. From 3.
5. $|AM|=|f(A)f(M)|=|A'M'|$.	5. Isometry preserves distances.

8. Prove that if $\triangle ABC\cong\triangle A'B'C'$, then the lengths of corresponding angle bisectors are equal.

Proof:

1. $\triangle ABC\cong\triangle A'B'C'$.	1. Given.
2. $\angle A=\angle A'$, $\angle C=\angle C'$, $|AC|=|A'C'|$.	2. From 1.
Let CD and C'D' be angel bisectors.	
3. $\angle ACD=1/2\angle C$.	3. From the definition of the angle bisector.
4. $\angle A'C'D'=1/2\angle C'$.	4. From the definition of the angle bisector.
5. $\angle ACD=\angle A'C'D'$.	5. From 2.,3. and 4.
6. $\triangle ACD\cong\triangle A'C'D'$.	6. ASA ($\angle A=\angle A'$, $|AB|=|A'B'|$, $\angle ACD=\angle A'C'D'$, from statements 2. and 5.)
7. $|CD|=|C'D'|$.	7. From 6.