Problem 2. Prove using two column form SAS theorem.

Given: $\triangle ABC, \triangle A'B'C'$, $\angle A=\angle A'$, $|AB|=|A'B'|, |AC|=|A'C'|$.
Prove: there exists an isometry such that f(A)=A', f(B)=B', f(C)=C'.

Analysis: $\angle A=\angle A'$, so from the Theorem 15c, we know that they are congruent and we know that we can construct an isometry that maps A to A' and the ray AB to the ray A'B, the ray AC to the ray A'C'. But why f(B)=B' and f(C)=C'? What do we know about f(B) and B'? We know that the distance between f(B) and A'=f(A) is |AB| and we know that |AB|=|A'B'|. It means that both points f(B) and B' lie on the same circle centered at A'. But we also know that both points lie on the same ray A'B'. But there is only one point of intersection of the ray A'B' and the circle centered at A' (Theorem 14), hence f(B)=B'.

Proof:

Statements:	
1. $\angle A=\angle A'$.	1. Given.
2. There exists an isometry f that maps $\angle A$ onto $\angle A'$, such that f(A)=A', $f(\overrightarrow{AB})=\overrightarrow{A'B'}$ and $f(\overrightarrow{AC})=\overrightarrow{A'C'}$.	2. Theorem 15c.
3. Denote f(B) as B", then B" lies on the ray $\overrightarrow{A'B'}$.	3. Statement 2.
4. B' lies on the ray A'B'	4. Given.
5. |B"A'|=|f(B)f(A)|=|AB|	5. Isometry preserves distances.
6. |B'A'|=|AB|.	6. Given.
7. Points B' and B" belong to the circle centered at A' and the radius |AB|.	7. Statement 4 and 5 and the definition of this circle.
8. Points B' and B" belong to the intersection of the circle centered at A' with the radius |AB| and the ray $\overrightarrow{A'B'}$.	8. Statements 3,4 and 7.
9. There is only one point of the intersection of the circle centered at A' with the radius |AB| and the ray $\overrightarrow{A'B'}$.	9. Theorem 14.
10. B'=B"=f(B)	10. Statements 8 and 9.
11. C'=f(C)	11. The same as 10.