Solutions.

Problem 1. Prove using two column form that isometries map lines to lines.

In order to solve this problem we will fix an isometry f and two points A and B on a line, then show that
1) every point on the line AB is mapped to a point on the line through f(A) and f(B)
2) every point on the line through f(A) and f(B) is the image of some point on the line AB.
If we prove these two statements then the image of the line AB will be the line f(A)f(B)

Proof: Statements:	Reason:
1. Fix two different points on the line. Call them A and B.	1. By Theorem 3, lines always have two points.
2. Choose any point X on the line $\overleftrightarrow{AB}$. One of the points X,A,B is between the other two. Three cases are possible: a) A is between X and B. b) X is between A and B. c) B is between X and A.	2. Axiom 3.2.
3. Assume f is an isometry, then one of the points f(X),f(A) and f(B) is between the other two.	3. Theorem 5.
4. Points f(X),f(A) and f(B) are on the same line.	4. Axiom 3.2.
5. $f(A)\neq f(B)$.	5. Isometry maps different point to different points, Theorem 4(isometries are bijections).
6. There is only one line through points f(A) and f(B). We will denote it as $\overleftrightarrow{f(A)f(B)}$.	6. Axiom 1.2.
7. If a point X is on the line $\overleftrightarrow{AB}$, then the point f(X) is on the line $\overleftrightarrow{f(A)f(B)}$.	7. We know from the statement 4, that points f(X),f(A) and f(B) are on the same line. But we know from the statement 6, that there is only one line that contains f(A) and f(B), hence the lines from statement 4 and 6 are the same line, and f(X) is on the line $\overleftrightarrow{f(A)f(B)}$.
8. The image of the line $\overleftrightarrow{AB}$ is a subset of the line $\overleftrightarrow{f(A)f(B)}$.	7. It is reformulation of the statement 7.
9. If a point Y is on the line $\overleftrightarrow{f(A)f(B)}$, then one of the points Y,f(A),f(B) is between the other two.	9. Axiom 3.2.
10. f^{-1} is an isometry.	10. Theorem 4.
11. If a point Y is on the line $\overleftrightarrow{f(A)f(B)}$, then one of the points $f^{-1}(Y)=X, f^{-1}f(A)=A, f^{-1}f(B)=B$ is between the other two.	11. Statement 9 and Theorem 5.
12. Points X,A and B are on the same line.	12. Axiom 3.2.
13. There is only one through points A and B.	13. Axiom 1.2.
14. The point X is on the line $\overleftrightarrow{AB}$.	14. Statement 12 and 13.
15. Every point Y on the line $\overleftrightarrow{f(A)f(B)}$ is the image some point from the the line $\overleftrightarrow{AB}$.	15. Y=f(X) and we know from the statement 14 that X is on the line $\overleftrightarrow{AB}$.
16. Line $\overleftrightarrow{f(A)f(B)}$ is a subset of the image of the line $\overleftrightarrow{AB}$.	16. Reformulation of the statement 15.
17. Line $\overleftrightarrow{f(A)f(B)}$ is the image of the line $\overleftrightarrow{AB}$.	17. Statement 8 and 16.