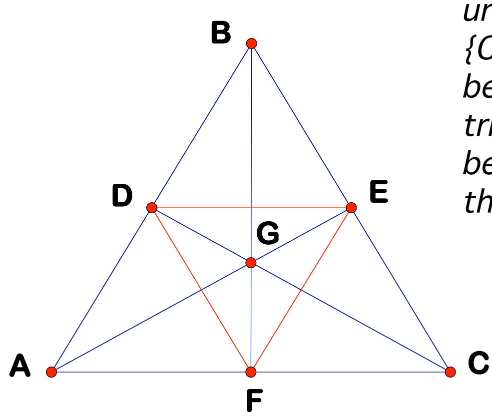
Problem 1.

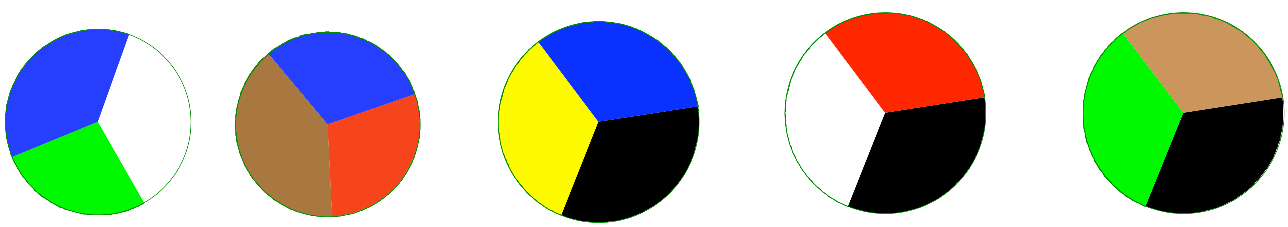
We introduce three basic notions: points, lines, incidence
They satisfy the following axioms:
1. For every two points there is exactly one line incident with both.
2. For every two lines there is exactly one point incident with both.
3. There exist four points such that no three of which are incident with a line.
4. Every line is incident with exactly three points.

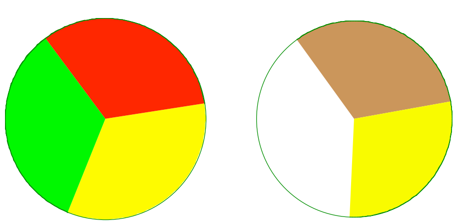
a) Prove that there are at least seven points in this geometry
b)(extra credit) Prove that there are exactly seven points in this geometry

Problem 2.



In this example the points are usual points (red dots). There are seven of them. Lines consist from unions of three points: {ADB}, {FGB}, {CEB},{AFC}, {CGD}, {AGE} and {FDE}. These are the points that belong to each of blue segments and to the red triangle. Points incident with a line means that they belong to the line. Lines incident with a point means the point belongs to each line.





In this model points are balls, lines are colors. Points incident with a line, if this color could be found on all balls. Lines are incident with a ball, if all these colors could be found on the ball.

Assign to each point a ball and to each line a color in such way that it preserves the incidence relation. It means if points incident with a line that corresponding balls are incident with the corresponding color and if lines incident with a point then the corresponding colors must be incident with the corrseponding ball.