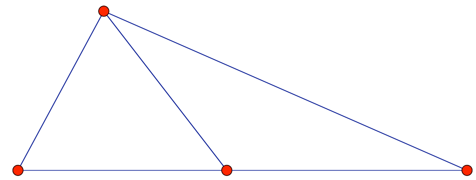
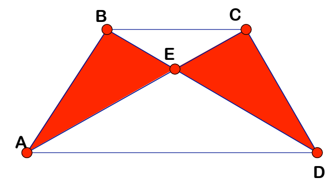
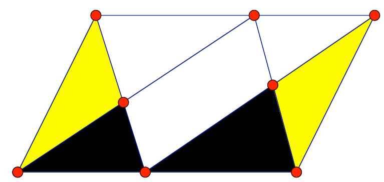
Important fact: if two triangles have a common vertex and sides oppozite to this vertex lie on the same line, then the ratio of the areas is equal to the ratio of the sides. If the sides are equal, then the areas are also equal.



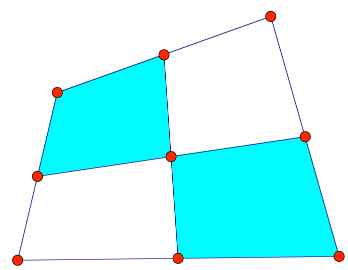
Problem 51. In a trapezoid ABCD AD||BC. We denote the point of intersection of the diagonals as P. Prove, that Area(△APB)=Area(△DPC).



Problem 52. Parallelogram is divided in 6 triangles and one quadrilateral, as shown in the picture. Prove that the sum of areas of two black triangles is equal to the sum of areas of two white triangles. Prove that the area of the quadrilateral is equal to the sum of the areas of two yellow triangles.

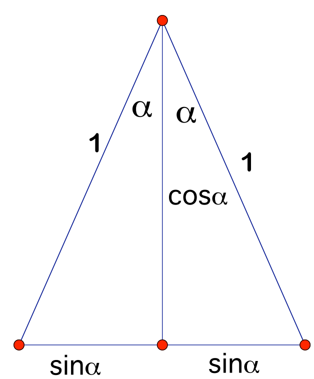


Problem 53. In convex quadrilateral the midpoints of opposite sides are connected by lines. These lines divide the quadrilateral in four parts. Prove that the sum of the areas of two opposite sides is equal to the sum of the areas of the other two opposite parts.



Problem 54. Prove using areas that if CD is the angle bisector of the triangle ABC, then

$$\frac{|AD|}{|DB|}=\frac{|AC|}{|BC|}$$



Problem 55. Prove that $\sin 2\alpha = 2\sin\alpha\cos\alpha$.

Solution:

$$S=\frac{1}{2}ha=\frac{1}{2}\cos\alpha\sin\alpha \text{ and } S=\frac{1}{2}ab\sin\angle C=\frac{1}{2}1\cdot 1\cdot \sin 2\alpha.$$

Just compare these two equalities.