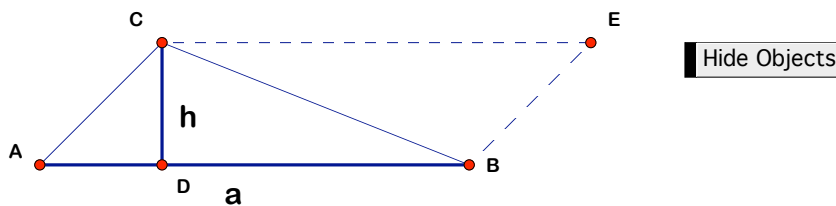


Theorem 62. The area of a triangle is  $\frac{1}{2}h \cdot a$ , where  $a$  is a side and  $h$  is the altitude to this side.

Proof:



ACEB is a parallelogram, therefore  $|CE|=|AB|$ ,  $|AC|=|BE|$ , hence  $\triangle ABC \cong \triangle ECB$  by SSS. Then we see that  $h \cdot a = \text{Area}(\text{ACEB}) = 2\text{Area}(\triangle ABC)$ . It follows that

$$\text{Area}(\triangle ABC) = \frac{1}{2}h \cdot a$$

Theorem 63. The area of a triangle is  $\frac{1}{2}a \cdot b \cdot \sin \alpha$ , where  $a$  and  $b$  are the sides and  $\alpha$  is the angle between them.

Proof:  $h = b \cdot \sin \angle A$ , so  $\frac{1}{2}h \cdot a = \frac{1}{2}a \cdot b \cdot \sin \alpha$ .