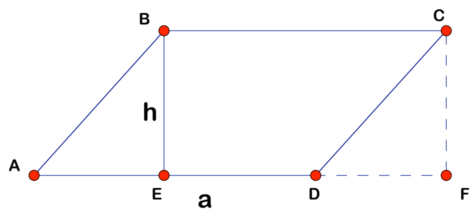
Theorem 60. The area of a parallelogram is $h \cdot a$, where a is a side and h is the altitude to this side.

Proof:

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Draw a perpendicular from the point C to the line AD.

| | |
|---|---|
| 1. $BE \parallel CF$. | 1. Theorem 18. They are $\perp$ to AF. |
| 2. $BC \parallel EF$. | 2. Given. |
| 3. EBCF is a parallelogram. | 3. From 1. and 2. |
| **4. $\lvert BE\rvert=\lvert CF\rvert=h$.** | 4. Opposite sides of a parallelogram are equal. |
| 5. $\lvert AB\rvert=\lvert DC\rvert$. | 5. Opposite sides of a parallelogram are equal. |
| **6. $\triangle ABE \equiv \triangle DCF$.** | 6. 4.,5. and HL. |
| 7. Area($\triangle$ABE)=Area($\triangle$DCF). | 7. Property 3 of the areas. |
| 8. Area(ABCD)=Area($\triangle$ABE)+Area(EBCD)= Area($\triangle$DCF)+Area(EBCD)=Area(EBCF). | 8. Property 2 of the areas and 7. |
| 9. EBCF is a rectangle. | 9. EBCF is a parallelogram(3.) and $\angle E=90°$. |
| 10. $\lvert AD\rvert=\lvert DF\rvert$. | 10. From 6, corresponding sides are equal. |
| 11. $\lvert AD\rvert=\lvert AE\rvert+\lvert ED\rvert=\lvert DF\rvert+\lvert ED\rvert=\lvert EF\rvert$. | |
| 12. Area(ABCD)=Area(EBCF)=ha. | 12. Theorem 59. |

Theorem 61. The area of a parallelogram is $a \cdot b \cdot \sin\alpha$, where a and b are the sides and $\alpha$ is the angle between them.

Proof: $h=\lvert AB\rvert \cdot \sin\angle A$, so Area(ABCD)$=\lvert AB\rvert \cdot a \cdot \sin\angle A$.