

Definition. Area is a function that assigns numbers to shapes, such that

1. $S(A) \geq 0$.
2. If a shape C is divided in two parts A and B then
$$S(C) = S(A) + S(B).$$
3. If a shape A is congruent to a shape B then
$$S(A) = S(B).$$
4. If A is a square with a side 1, then
$$S(A) = 1.$$

Theorem 58 (Area of a square).

a) Area of a square with a side n is n^2 .

Proof: Divide a square with a side n in n^2 squares with sides 1. All of them are congruent to each other. If we denote the area of a square with a side n as x , then from 2. follows that

$$n^2 \cdot 1 = x \text{ or that } x = n^2.$$

b) Area of a square with a side $\frac{1}{n}$ is $\frac{1}{n^2}$.

Proof: Divide a square with a side 1 in n^2 squares with sides $\frac{1}{n}$. All of them are congruent to each other. If we denote the area of a square with a side $\frac{1}{n}$ as x , then from 2. follows that

$$n^2 \cdot x = 1 \text{ or that } x = \frac{1}{n^2}.$$