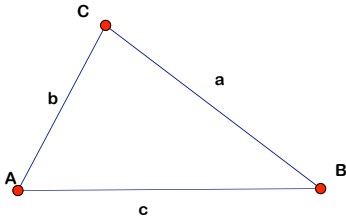


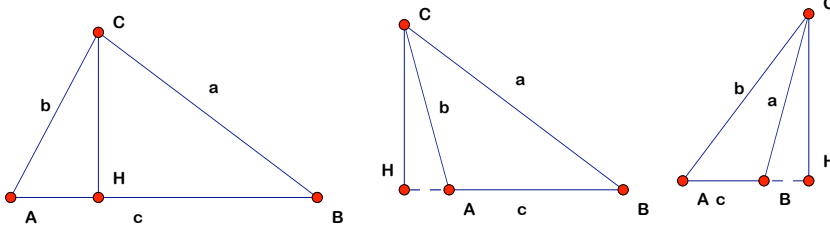
Theorem 56 (cos theorem). In a $\triangle ABC$, $|AB|=c, |AC|=b$ and $|BC|=a$. Then

$$a^2=c^2+b^2-2cb\cdot\cos\angle A$$



Proof:

Draw altitude from C to AB. There are three cases:



1. $|CH|=b\cdot\sin\angle A$ in all three cases.

$|AH|=b\cdot\cos\angle A$ in the first and the third case and $|AH|=-b\cdot\cos\angle A$ in the second case.

2. In the first case $|HB|=c-b\cdot\cos\angle A$,
in the second case $|HB|=c+b\cdot\cos\angle A=c-b\cdot\cos\angle A$,
in the third case $|HB|=b\cdot\cos\angle A-c$

3. $|HB|=|c-b\cdot\cos\angle A|$

$$\begin{aligned} 4. a^2 &= |CH|^2 + |HB|^2 = b^2 \cdot \sin^2 \angle A + (c - b \cdot \cos \angle A)^2 = \\ &= b^2 \cdot \sin^2 \angle A + b^2 \cdot \cos^2 \angle A - 2bc \cdot \cos \angle A + c^2 = \\ &= b^2 (\sin^2 \angle A + \cos^2 \angle A) + c^2 - 2bc \cdot \cos \angle A = \\ &= b^2 + c^2 - 2bc \cdot \cos \angle A \end{aligned}$$

1. From definition of sin and cos.

In the second case the angle is $>90^\circ$ and cos is negative.

2. Follows from pictures.

3. Follows from 2.

4. Pythagorean Theorem