

Theorem 46. (SSS for similar triangles). If corresponding sides of two triangles are proportional then they are similar and their corresponding angles are equal.

Proof: We denote triangles as  $\triangle ABC$  and  $\triangle A'B'C'$ . We assume that  $\frac{|A'B'|}{|AB|} = \frac{|B'C'|}{|BC|} = \frac{|A'C'|}{|AC|} = k$ . Consider a central dilation  $D_{O,k}$  with the factor  $k$ . We denote the image of the  $\triangle ABC$  under this dilation as  $\triangle A''B''C''$ .

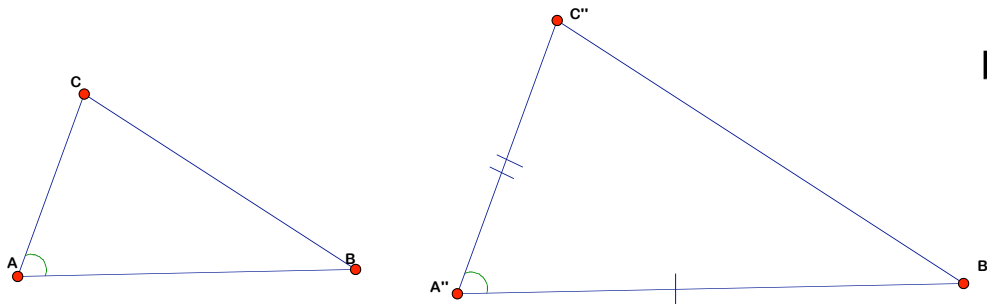
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| 1. $\triangle ABC \sim \triangle A''B''C''$ .                                    | 1. Given ( $\triangle A''B''C''$ is the image of the $\triangle ABC$ under a dilation) |
| 2. $\frac{ A''B'' }{ AB } = \frac{ B''C'' }{ BC } = \frac{ A''C'' }{ AC } = k$ . | 2. From 1. .   |
| 3. $\frac{ A'B' }{ AB } = \frac{ B'C' }{ BC } = \frac{ A'C' }{ AC } = k$         | 3. Given.  |
| 4. $ A''B''  =  A'B' $ , $ B''C''  =  B'C' $ , $ A''C''  =  A'C' $ .             | 4. 2. and 3. .   |
| 5. $\triangle A''B''C'' \cong \triangle A'B'C'$ .                                | 5. 4. and SSS.   |
| 6. $\triangle ABC \sim \triangle A'B'C'$ .                                       | 6. 1. and 6..  |

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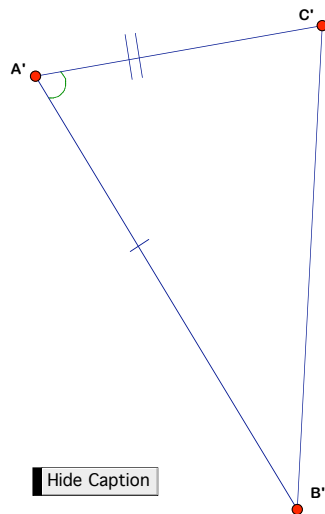
Theorem 47. (SAS for similar  $\triangle$ s). If  $\frac{|A'B'|}{|AB|} = \frac{|A'C'|}{|AC|}$  and  $\angle A = \angle A'$ , then  $\triangle ABC \sim \triangle A'B'C'$ .

Proof: We denote triangles as  $\triangle ABC$  and  $\triangle A'B'C'$ . We assume that  $\frac{|A'B'|}{|AB|} = \frac{|A'C'|}{|AC|} = k$ . Consider a central dilation  $D_{O,k}$  with the factor  $k$ . We denote the image of the  $\triangle ABC$  under this dilation as  $\triangle A''B''C''$ .



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- $\triangle ABC \sim \triangle A''B''C''$ .
- $\frac{|A''B''|}{|AB|} = k$ ,  $\frac{|A''C''|}{|AC|} = k$
- $\frac{|A'B'|}{|AB|} = k$ ,  $\frac{|A'C'|}{|AC|} = k$
- $|A''B''| = |A'B'|$ ,  $|A''C''| = |A'C'|$ .
- $\angle A = \angle A'$ .
- $\angle A = \angle A''$ .
- $\angle A'' = \angle A'$ .
- $\triangle A''B''C'' \cong \triangle A'B'C'$ .
- $\triangle ABC \sim \triangle A'B'C'$ .



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- $\triangle A''B''C''$  is the image of  $\triangle ABC$  under a dilation with factor  $k$ .
- Follows from 1..
- Given.
- From 2. and 3..
- Given.
- Follows from 1..
- From 6. and 7.
- SAS, using 4. and 7.
- From 1. and 8..