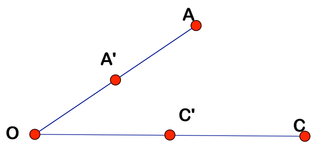
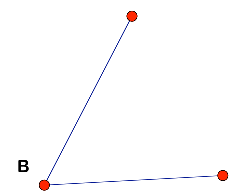
Theorem 42. If f is a central dilation with the center O, then f preserves angle with vertex at O. In other words, if f is a dilation with the center at the point O, then $\angle AOC = \angle f(A)f(O)f(C)$.



Question: What can you say about a composition of $D_{O,k}$ and $D_{O,1/k}$?

Question: What can you say about a composition of a dilation with factor k and $D_{O,1/k}$?

Theorem 43. Dilations preserve angles. In other words, if f is a dilation then $\angle ABC = \angle f(A)f(B)f(C)$.



Proof: $f = fD_{B,1/k}D_{B,k} = (fD_{B,1/k})D_{B,k} = gD_{B,k}$, where g is isometry. Isometry preserves angles, $D_{B,k}$ preserves angles centered at B, hence their composition preserves angles centered at B.

Definition: two sets are called *similar* if there exists a dilation that maps one set onto the other.

Similarity is relation of equivalence.

1. A~A.
2. If A~B, then B~A.
3. If A~B and B~C, then A~C.

Theorem 44. If two polygons are similar, then their corresponding sides are proportional and their corresponding angles are equal.
Proof: Follows from the definition of similar maps and the Theorem 43.

Theorem 45. If two polygons $A_1, A_2, \ldots, A_n$ and $B_1, B_2, \ldots, B_n$, have proportional sides:

$$\frac{|A_1A_2|}{|B_1B_2|} = \ldots = \frac{|A_{n-1}A_n|}{|B_{n-1}B_n|} = \frac{|A_nA_1|}{|B_nB_1|} = k$$

and $\angle A_1 = \angle B_1, \ldots, \angle A_n = \angle B_n$ then polygons are similar.