Definition: a map is called a *dilation* with factor k>0 if for any two points A and B the following is true:

$$\text{dist}(f(A),f(B))=k\cdot\text{dist}(A,B).$$

Another way to write it:

$$|f(A)f(B)|=k\cdot|AB|.$$

Theorem 40.

a) Identity is a dilation.

b) If f is a dilation with factor k, then f is invertible and f^{-1} is a dilation with factor 1/k.

Hide Caption

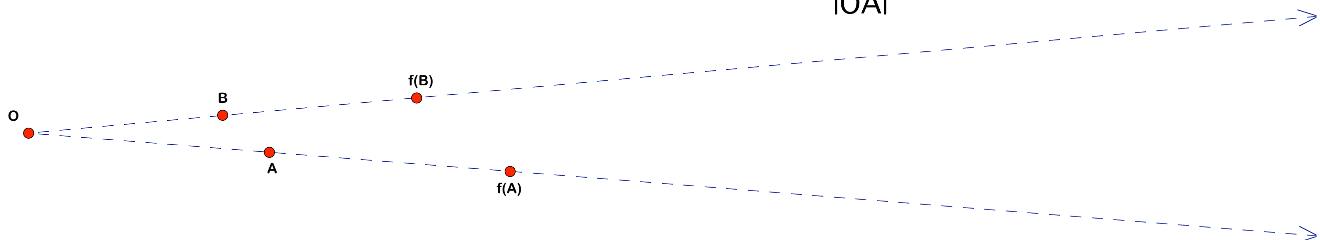
c) A composition of two dilations with factors k and l is a dilation with factor k·l.

d) Dilations maps lines to lines.

Proof is easy.

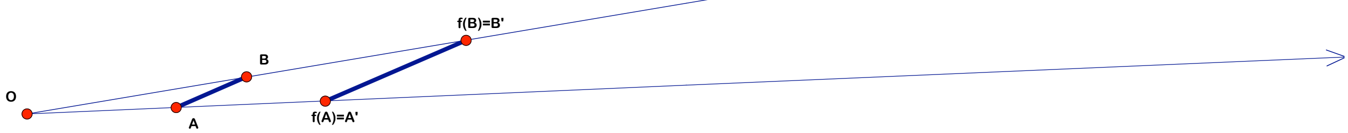
Definition: a map is called a *central dilation* with *center* O and factor k>0 if it maps a point A to the point A', such that A' is on the ray $\overrightarrow{OA}$ and $\frac{|OA'|}{|OA|}=k$.

Hide Objects



Theorem 41. Central dilation is a dilation.

Let f is a central dilation with factor k and the center O. We denote f(A) as A', and f(B) as B'.



Proof:

Statements:	Reasons:
1. $\frac{|OA|}{|OA'|}=k$	1. Given.
2. $\frac{|OB|}{|OB'|}=k$	2. Given.
3. $\frac{|OA|}{|OA'|}=\frac{|OB|}{|OB'|}=k$	3. From 1. and 2.
4. BA||B'A'	4. Statement 3 and Theorem 36.
5. $\frac{|BA|}{|B'A'|}=\frac{|OA|}{|OA'|}=k$	5. Statement 4 and Theorem 37.
6. dist(f(B),f(A))=|B'A'|=k·|BA|=dist(B,A)	6. From statment 5.
7. f is dilation.	7. From 6 and the definition of dilations.