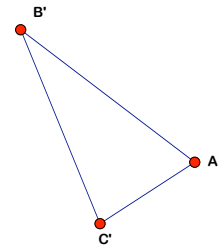
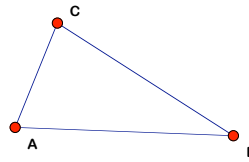
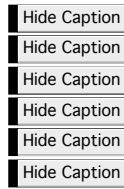


Theorem 16.(ASA) If two triangles have two angles and the side between them equal, then triangles are congruent.



Denote the triangles as $\triangle ABC$ and $\triangle A'B'C'$. We assume that $\angle A = \angle A'$, $\angle B = \angle B'$ and $|AB| = |A'B'|$.

Proof:

Reasons:

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| <ol style="list-style-type: none"> 1. There exists an isometry f that maps $\angle A$ onto $\angle A'$, moreover ray AB onto $A'B'$, ray AC onto $A'C'$. 2. $f(A) = A'$ 3. $A'B' = A'f(B)$ 4. $f(B) = B'$ 5. Rays BC and AC lie on the same of the line AB. 6. Rays $f(BC) = B'f(C)$ and $f(AC) = A'f(C)$ lie on the same side of $A'B'$. 7. Rays $B'f(C)$ and $A'C'$ lie on the same side of $A'B'$. 8. Rays $B'f(C)$ and $A'C'$ lie on the same side of $A'B'$. 9. $\angle ABC = \angle A'B'C'$ 10. $\angle A'B'f(C) = \angle A'B'C'$ 11. Ray $f(BC) = \text{ray } B'C'$. 5. $f(C) = C'$ | <ol style="list-style-type: none"> 1. Theorem 15, part c). 2. The point A is the intersection of the lines AB and AC. They are mapped to lines $A'B'$ and $A'C'$. The point of intersection of AB and AC is mapped to the point of intersection of $A'B'$ and $A'C'$. 3. $A'B' = AB = f(A)f(B) = A'f(B)$ 4. From the statement 3. and Theorem 14 follows that points $f(B)$ and B' are the same point. 5. They lie on the same side as the point C. 6. By Theorem 8, halfplanes are mapped to halfplanes. By statement 5 BC and AC are in the same halfplane with AB as the boundary. Hence, their images are also in the same halfplane with the boundary $f(AB) = A'B'$ 7. It is given to us that the $f(AC) = A'C'$. 8. It follows from statements 6 and 7. 9. It is given. 10. $\angle A'B'C' = \angle ABC = f(\angle ABC) = \angle A'B'f(C)$ 11. $\angle A'B'C' = \angle A'B'f(C)$ and rays $B'C'$ and $B'f(C)$ on the side of $A'B'$. We can apply Corollary 15.1. 12. The point C is the intersection of the lines AC and BC. Hence $f(C)$ is the intersection of the lines $f(AC)$ and $f(BC)$. But lines $f(AC) = A'C'$ and $f(BC) = B'C'$. They intersect at the point C'. Hence $f(C) = C'$. |
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