

Theorem 7

Proof, Th 7

Theorem 8

Halfplane

Axiom 4.1

Definition.

Two lines are called *parallel* if they don't intersect.

Theorem 8. If l is a line and h_1 and h_2 are two halfplanes with l as the boundary then $f(h_1)$ and $f(h_2)$ are halfplanes with $f(l)$ as their boundary.

Theorem 7. Isometry maps intersecting sets to intersecting sets, disjoint sets to disjoint sets. In particular, two lines are parallel if and only if their images under an isometry are parallel lines. Line and a segment intersect if and only if their images intersect

Proof: If f is invertible map then it maps disjoint sets to disjoint sets. Hence, if lines did not intersect each other their images under an isometry remain disjoint.

4. Axioms about Isometries.

1. If $|AB|=|CD|$ then there exists exactly two isometries such that $f(A)=C$, $f(B)=D$. Moreover if α is one of the halfplanes with the line AB as a boundary, β and γ are halfplanes with the line CD as the boundary then one of those isometries maps α to β and the other maps α to γ .