

Theorem 6. Isometries map segments to segments, lines to lines.

Proof:

Assume AB is a segment.

Statement:

1. A segment AB consists from points that are between A and B .

2. If a point X is between A and B then point $f(X)$ is between $f(A)$ and $f(B)$.

3. If point X belongs to the segment AB then $f(X)$ belongs to the segment $f(A)f(B)$, hence $f(AB)$ is a subset of $f(A)f(B)$.

4. If a point Y is between $f(A)$ and $f(B)$ then $f^{-1}(Y)$ is between A and B

5. If Y is a point that belongs to the segment $f(A)f(B)$ then there exists a point X that belongs to AB such that $Y=f(X)$, hence $f(A)f(B)$ is a subset of $f(AB)$.

6. Segment $f(A)f(B)$ coincides with the image of the segment AB under the isometry f .

Reasons:

1. The definition of a segment.

2. Theorem 5 proves that.

3. From 2. follows that $f(X)$ is between $f(A)$ and $f(B)$, hence from the definition of a segment follows that $f(X)$ belongs to the segment $f(A)f(B)$.

4. Theorem 4 tells us that f^{-1} is an isometry. Theorem 5 tells that $f^{-1}(Y)$ is between $f^{-1}(f(A))=A$ and $f^{-1}(f(B))=B$

5. Take $X=f^{-1}(Y)$. Then X is between A and B (statement 5), hence belongs to the segment AB and $f(X)=f(f^{-1}(Y))=Y$.

6. It follows from statements 3 and 5.

1	r1
2	r2
3	r3
4	r4
5	r5
6	r6