Quiz 2 (10 pts)
Math 220, Spring 2008
Due Tuesday, February 19.

NAME:
STUDENT ID NUMBER:

SHOW YOUR WORK!

Problem 1. (5pts) a) Describe all solutions of \( A\vec{x} = \vec{0} \) in parametric form, where \( A \) is following matrix:

\[
A = \begin{bmatrix}
1 & 3 & -3 & 7 \\
2 & 6 & -10 & 19
\end{bmatrix}
\]

\[
\begin{bmatrix}
1 & 3 & -3 & 7 \\
2 & 6 & -10 & 19
\end{bmatrix} \iff \begin{bmatrix}
1 & 3 & -3 & 7 \\
0 & 0 & -4 & 5
\end{bmatrix} \iff \begin{bmatrix}
1 & 3 & 0 & \frac{13}{4} \\
0 & 0 & 1 & -\frac{5}{4}
\end{bmatrix}
\]

\[
x_1 = -3x_2 - \frac{13}{4}x_4 \\
x_2 = x_2 \\
x_3 = \frac{5}{4}x_4 \\
x_4 = x_4
\]

\[
\vec{x} = x_2 \begin{bmatrix}
-3 \\
1 \\
0 \\
0
\end{bmatrix} + x_4 \begin{bmatrix}
-\frac{13}{4} \\
0 \\
\frac{5}{4} \\
1
\end{bmatrix}
\]

b) How geometrically the set of solutions looks like? Possible answers are: a point, a line, a plane, a 3d space.

It is a homogeneous system of linear equations with two free variables, so the set of solutions is a plane that passes through the origin.
Problem 2. (5pts) a) Is \( \vec{u} \) in the plane \( \mathbb{R}^2 \) spanned by the columns of \( A \)? Explain why or why not. If you only write yes or no without a solution you will get 0 points.

\[
\vec{u} = \begin{bmatrix} 0 \\ 4 \\ 4 \end{bmatrix}, \quad A = \begin{pmatrix} 3 & -5 \\ -2 & 6 \\ 1 & 1 \end{pmatrix}
\]

A vector \( \vec{u} \) belongs to the span of the rows of the matrix \( A \) that we denote as \( \vec{v}_1 \) and \( \vec{v}_2 \) if we can find weights \( x_1, x_2 \) such that

\[
x_1 \vec{v}_1 + x_2 \vec{v}_2 = \vec{u}
\]

In other words \( \vec{u} \) belongs to the span if the system of linear equations \( A\vec{x} = \vec{u} \) is consistent (we can find weights).

\[
\begin{bmatrix} 3 & -5 & 0 \\ -2 & 6 & 4 \\ 1 & 1 & 4 \end{bmatrix} \iff \begin{bmatrix} 1 & 1 & 4 \\ 3 & -5 & 0 \\ -2 & 6 & 4 \end{bmatrix} \iff \begin{bmatrix} 1 & 1 & 4 \\ 0 & -8 & 12 \\ 0 & 0 & 0 \end{bmatrix} \iff \begin{bmatrix} 1 & 1 & 4 \\ 0 & -8 & -12 \\ 0 & 0 & 0 \end{bmatrix}
\]

The system is consistent, hence the vector \( \vec{u} \) is in the span.

b) Find what condition coordinates \( b_1, b_2 \) and \( b_3 \) must satisfy so the vector \( [b_1, b_2, b_3] \) was in the span of the columns of the matrix \( A \) from the part a).

We need to check when the system of linear equations \( A\vec{x} = \vec{b} \) is consistent.

\[
\begin{bmatrix} 3 & -5 & b_1 \\ -2 & 6 & b_2 \\ 1 & 1 & b_3 \end{bmatrix} \iff \begin{bmatrix} 1 & 1 & b_3 \\ 3 & -5 & b_1 \\ -2 & 6 & b_2 \end{bmatrix} \iff \begin{bmatrix} 1 & 1 & b_3 \\ 0 & -8 & b_1 - 3b_3 \\ 0 & 0 & b_2 + 2b_3 \end{bmatrix} \iff \begin{bmatrix} 1 & 1 & b_3 \\ 0 & -8 & b_1 - 3b_3 \\ 0 & 0 & b_2 + 2b_3 + (b_1 - 3b_3) \end{bmatrix}
\]

The system is consistent if and only if \( b_2 + 2b_3 + (b_1 - 3b_3) = 0 \) or

\[
b_1 + b_2 - b_3 = 0.
\]