MATH 220 MIDTERM EXAMINATION March 5, 2007

Name______________________________ ID #__________________ Section #______

There are 20 multiple choice questions. Each problem is worth 5 points. Four possible answers are given for each problem, only one of which is correct. When you solve a problem, note the letter next to the answer that you wish to give and blacken the corresponding space on the answer sheet. **Mark only one choice; darken the circle completely** (you should not be able to see the letter after you have darkened the circle).

THE USE OF CALCULATORS DURING THE EXAMINATION IS FORBIDDEN.

PLEASE SHOW YOUR PSU ID CARD TO THE PROCTOR WHEN YOU FINISH.

CHECK THE EXAMINATION BOOKLET BEFORE YOU START. THERE SHOULD BE 20 PROBLEMS ON 11 PAGES (INCLUDING THIS ONE).

1. Find the solution to the following linear system:

$$\begin{array}{rcrcrcl} x_1 & - & 2x_2 & + & 4x_3 & = & 1 \\ 3x_1 & - & 8x_2 & + & 10x_3 & = & 7 \\ 2x_1 & - & 3x_2 & + & 9x_3 & = & 0 \end{array}.$$

a) $x_1 = -3, \quad x_2$ is free, $\quad x_3 = -2 + x_2$.

b) $x_1 = -3 - 6x_3, \quad x_2 = -2 - x_3, \quad x_3$ is free.

c) $x_1 = -9, \quad x_2 = -3, \quad x_3 = -1$.

d) There is no solution.

2. Which of the following statements is **always** true?

a) A consistent linear system has only one solution.

b) If a homogeneous linear system has at least one free variable, then the system has many solutions.

c) The augmented matrix of a linear system has a unique echelon form.

d) If an echelon form of the augmented matrix of a linear system has a zero row, then the system is consistent.

3. Let $\begin{bmatrix} 1 & -1 & 2 & 2 \\ 1 & -3 & 1 & k \\ 3 & 1 & -2h & 5 \end{bmatrix}$ be the augmented matrix of a linear system. For which value(s) of h, k is the system inconsistent?

a) $h = -4,\ k = \frac{5}{2}$.

b) $h \neq -4,\ k = \frac{5}{2}$.

c) $h = -4,\ k \neq \frac{5}{2}$.

d) $h \neq -4,\ k \neq \frac{5}{2}$.

4. Which of the following matrices is in reduced echelon form?

a) $\begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

b) $\begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

c) $\begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

d) $\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

5. Let $\mathbf{u} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} -1 \\ 2 \\ 0 \end{bmatrix}$. Which of the following vectors is in the span of $\{\mathbf{u}, \mathbf{v}\}$?

a) $\begin{bmatrix} 3 \\ 4 \\ -1 \end{bmatrix}$

b) $\begin{bmatrix} -1 \\ 3 \\ 0 \end{bmatrix}$

c) $\begin{bmatrix} 3 \\ -2 \\ 2 \end{bmatrix}$

d) $\begin{bmatrix} -5 \\ -1 \\ 1 \end{bmatrix}$

6. Let $\begin{bmatrix} 1 & -2 & 0 & b_1 \\ 0 & 4 & 0 & b_1 + b_2 \\ 0 & 0 & b_2 & b_1 + 4b_2 + 2 \end{bmatrix}$ be the augmented matrix of a linear system, then which of the following conditions implies that the linear system is consistent?

a) $b_1 = 1$, $b_2 = 0$.

b) $b_1 \neq -2 - 4b_2$, $b_2 = 0$.

c) $b_1 = -2 - 4b_2$, $b_2 \neq 0$.

d) The system is always consistent.

7. Let $A = \begin{bmatrix} 2 & -4 & 4 \\ 0 & 2 & 1 \\ -1 & 8 & 1 \end{bmatrix}$, then which of the following is the solution of $A\mathbf{x} = \mathbf{0}$ in parametric vector form?

a) $\begin{bmatrix} 6 \\ 1 \\ -2 \end{bmatrix}$

b) $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

c) $x_3 \begin{bmatrix} -3 \\ -0.5 \\ 1 \end{bmatrix}, \quad x_3 \in \mathbb{R}$

d) $x_2 \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix}, \quad x_2, x_3 \in \mathbb{R}$

8. Let $A = \begin{bmatrix} 0 & 1 & -1 \\ 2 & 1 & 1 \end{bmatrix}$. Suppose that for some $\mathbf{b}$ in $\mathbb{R}^2$, $\mathbf{p} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$ is one particular solution to the nonhomogeneous equation $A\mathbf{x} = \mathbf{b}$. What is the general form of the solution to this equation ($A\mathbf{x} = \mathbf{b}$)?

a) $\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$.

b) $\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} + x_3 \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}, \quad x_3 \in \mathbb{R}.$

c) $x_3 \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \quad x_3 \in \mathbb{R}.$

d) $\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} + x_3 \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \quad x_3 \in \mathbb{R}.$

9. Which of the following statements is always **true**?

a) If every row of A has a pivot position, then $A\mathbf{x} = \mathbf{0}$ has only the trivial solution.

b) If $A\mathbf{x} = \mathbf{0}$ has nontrivial solutions, then $A\mathbf{x} = \mathbf{b}$ is consistent for any $\mathbf{b}$.

c) If the solution to $A\mathbf{x} = \mathbf{b}$ is not unique, then $A\mathbf{x} = \mathbf{0}$ has nontrivial solutions.

d) If the solution to $A\mathbf{x} = \mathbf{b}$ is unique, then $A\mathbf{x} = \mathbf{0}$ has no solution.

10. Which of the following sets of vectors is linearly independent?

a) $\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ -1 \end{bmatrix} \right\}$.

b) $\left\{ \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ -2 \\ 0 \end{bmatrix} \right\}$.

c) $\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ -3 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\}$.

d) $\left\{ \begin{bmatrix} 0 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 0 \\ -3 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\}$.

11. Let $A = \begin{bmatrix} 1 & 1 & -2 \\ 2 & 1 & -4 \\ -1 & 0 & 2 \end{bmatrix}$. Which of the following is a nontrivial soluton to $A\mathbf{x} = \mathbf{0}$?

a) $\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}$.

b) $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$.

c) $\begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}$.

d) $\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$.

12. Let $A = \begin{bmatrix} \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \end{bmatrix}$. What is the geometric interpretation of the transformation $T(\mathbf{x}) = A\mathbf{x}$? (Note that $\cos(\pi/4) = \frac{\sqrt{2}}{2}$.)

a) A rotation by $\frac{\pi}{4}$ about the origin in the counterclockwise direction.

b) A shearing in the x_1 direction by a factor of $\frac{\sqrt{2}}{2}$.

c) A contraction by the factor of $\frac{\sqrt{2}}{2}$.

d) A reflection in the x_1-axis combined with a contraction by $\frac{\sqrt{2}}{2}$.

13. Let T be a linear transformation, and let $\mathbf{u}$, $\mathbf{v}$ be two vectors satisfying $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$ and $T(\mathbf{v}) = \begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix}$. Then $T(3\mathbf{u} - \mathbf{v})$ is:

a) $\begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix}$.

b) $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$.

c) $\begin{bmatrix} 1 \\ -2 \\ -3 \end{bmatrix}$.

d) There is not enough information to compute $T(3\mathbf{u} - \mathbf{v})$.

14. Find the standard matrix of T where $T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} x_1 + x_3 \\ 2x_1 + x_3 \\ x_1 - 5x_2 \end{bmatrix}$.

a) $\begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 5 \end{bmatrix}$.

b) $\begin{bmatrix} 1 & 1 \\ 2 & 1 \\ 1 & -5 \end{bmatrix}$.

c) $\begin{bmatrix} 1 & 0 & 1 \\ 2 & 0 & 1 \\ 1 & -5 & 0 \end{bmatrix}$.

d) $\begin{bmatrix} 1 & 2 & 1 \\ 0 & 0 & -5 \\ 1 & 1 & 0 \end{bmatrix}$.

15. Consider $T(\mathbf{x}) = A\mathbf{x}$, where $A = \begin{bmatrix} 2 & 1 \\ 0 & -1 \\ 3 & 2 \end{bmatrix}$. Which of the following statements is true?

a) T is one-to-one and onto.

b) T is onto but not one-to-one.

c) T is one-to-one but not onto.

d) T is neither one-to-one nor onto.

16. Let T be the transformation defined by $T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, then which of the following vectors is in the range of T?

a) $\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$.

b) $\begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$.

c) $\begin{bmatrix} -1 \\ 0 \\ 0 \end{bmatrix}$.

d) $\begin{bmatrix} 0 \\ -1 \\ 0 \end{bmatrix}$.