MATH 220

MIDTERM EXAM

OCTOBER 15, 2007

FORM A

NAME ____________________________

STUDENT NUMBER ____________________

INSTRUCTOR ________________________

SECTION NUMBER ____________________

This examination will be machine processed by the University Testing Service. Use only a number 2 pencil on your answer sheet. On your answer sheet, identify your name, this course (MATH 220) and the date. Code and blacken the corresponding circles on your answer sheet for your student I.D. number and the class section number. Code in your test form. **Five points will be deducted from the score of those who has forgotten to code in their student I.D. numbers or test forms.**

There are 20 multiple choice questions each worth five points. For each problem, four possible answers are given, only one of which is correct. You should solve the problem, note the letter of the answer that you wish to give and **blacken** the corresponding space on the **answer sheet**. Mark only one choice; darken the circle completely (you should not be able to see the letter after you have darkened the circle). Check frequently to be sure the problem number on the test sheet is the same as the problem number of the answer sheet.

**THE USE OF A CALCULATOR, CELL PHONE, OR ANY OTHER ELECTRONIC DEVICE IS NOT PERMITTED DURING THIS EXAMINATION.**

**CHECK THE EXAMINATION BOOKLET BEFORE YOU START. THERE SHOULD BE 20 PROBLEMS ON 12 PAGES (INCLUDING THIS ONE).**

1. Find all solutions to the following linear system:

$$\begin{array}{rrrcl} -x_1 & +2x_2 & +x_3 & = & 2 \\ x_1 & -2x_2 & +4x_3 & = & 9 \\ 2x_1 & -4x_2 & +3x_3 & = & 8 \end{array}$$

a) $x_1 = 1, x_2 = 0, x_3 = 2$

b) $x_1 = 2x_2, x_2$ is free , $x_3 = 2$

c) $x_1 = 2x_3, x_2 = 0, x_3$ is free

d) The system is inconsistent.

2. Which of the following matrices is in reduced echelon form?

a) $\begin{bmatrix} 1 & 0 & 3 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

b) $\begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$

c) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

d) $\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

3. If $A = \begin{bmatrix} 1 & h & 4 \\ 3 & 6 & 8 \end{bmatrix}$ is the augmented matrix for a system of linear equations, then for which values of $h$ is the system consistent?

a) $h = 2$
b) $h \neq 2$
c) The system is consistent for any value of $h$.
d) The system is inconsistent for any value of $h$.

4. If $\mathbf{v_1} = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$, $\mathbf{v_2} = \begin{bmatrix} 1 \\ 1 \\ 5 \end{bmatrix}$, $\mathbf{x} = 2\begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix} + 3\begin{bmatrix} 1 \\ 1 \\ 5 \end{bmatrix}$, $\mathbf{y} = \begin{bmatrix} 11 \\ 1 \\ 40 \end{bmatrix}$, then which of the following is true?

a) $\mathbf{v_1}$ is in Span$\{\mathbf{v_1}, \mathbf{v_2}\}$, and $\mathbf{x}$ is not in Span$\{\mathbf{v_1}, \mathbf{v_2}\}$.
b) $\mathbf{x}$ is in Span$\{\mathbf{v_1}, \mathbf{v_2}\}$, and $\mathbf{y}$ is in Span$\{\mathbf{v_1}, \mathbf{v_2}\}$.
c) $\mathbf{x}$ is not in Span$\{\mathbf{v_1}, \mathbf{v_2}\}$, and $\mathbf{y}$ is not in Span$\{\mathbf{v_1}, \mathbf{v_2}\}$.
d) $\mathbf{y}$ is in Span$\{\mathbf{v_1}, \mathbf{v_2}\}$, and $\mathbf{v_1}$ is not in Span$\{\mathbf{v_1}, \mathbf{v_2}\}$.

5. Let $A = \begin{bmatrix} 2 & -1 \\ -6 & 3 \end{bmatrix}$, and $\mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$. Describe the set of all $\mathbf{b}$ for which $A\mathbf{x} = \mathbf{b}$ is consistent.

a) $3b_1 + b_2 = 0$
b) $b_1 = 0$, $b_2 = 0$
c) $3b_1 + b_2 \neq 0$
d) All $b_1$ and $b_2$.

6. Let $A = \begin{bmatrix} 1 & 1 & -2 \\ 1 & 2 & 0 \\ 4 & 6 & -4 \end{bmatrix}$, then which of the following best describes the geometric form of the set of all solutions of $A\mathbf{x} = \mathbf{0}$?

a) It is the zero vector.
b) It is a line.
c) It is a plane.
d) It is a 3d-space.

7. If $A$ is an $m \times n$ matrix with columns $\mathbf{v_1}, \ldots, \mathbf{v_n}$, and $\mathbf{b}$ is a vector in $\mathbb{R}^n$ such that $A\mathbf{x} = \mathbf{b}$ has infinitely many solutions, then which of the following statements is false?

   a) $\mathbf{b}$ is in Span$\{\mathbf{v_1}, \ldots, \mathbf{v_n}\}$.
   b) The equation $x_1\mathbf{v_1} + \ldots + x_n\mathbf{v_n} = \mathbf{b}$ has a solution.
   c) The vectors $\{\mathbf{v_1}, \ldots, \mathbf{v_n}, \mathbf{b}\}$ are linearly independent.
   d) The system $A\mathbf{x} = \mathbf{b}$ has a free variable.

8. Which of the following sets of vectors is linearly independent?

   a) $\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ \sqrt{2} \end{bmatrix} \right\}$

   b) $\left\{ \begin{bmatrix} 5 \\ 7 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix} \right\}$

   c) $\left\{ \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 12 \\ 5 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} \right\}$

   d) $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \right\}$

9. Let $A = \begin{bmatrix} 1 & -4 & -2 & 0 & 3 & -5 \\ 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & -4 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$. Describe all solutions of $A\mathbf{x} = \mathbf{0}$ in parametric vector form.

a) $\mathbf{x} = x_6 \begin{bmatrix} 5 \\ 1 \\ 4 \\ 0 \\ 0 \\ 1 \end{bmatrix}$, with $x_3$ free.

b) $\mathbf{x} = x_2 \begin{bmatrix} 4 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_6 \begin{bmatrix} -5 \\ -1 \\ -4 \\ 0 \\ 0 \\ 1 \end{bmatrix}$, with $x_2$ and $x_6$ free.

c) $\mathbf{x} = x_2 \begin{bmatrix} 4 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_6 \begin{bmatrix} -5 \\ 0 \\ 1 \\ 0 \\ 4 \\ 1 \end{bmatrix}$, with $x_2$, $x_4$ and $x_6$ free.

d) $\mathbf{x} = x_2 \begin{bmatrix} 4 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + x_4 \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} + x_6 \begin{bmatrix} -5 \\ -1 \\ -4 \\ 0 \\ 0 \\ 1 \end{bmatrix}$, with $x_2$, $x_4$ and $x_6$ free.

10. Which of the following vectors satisfies the linear system $\begin{cases} 3x_1 + 5x_2 - 4x_3 &= 7 \\ -3x_1 - 2x_2 + 4x_3 &= -1 \\ 6x_1 + x_2 - 8x_3 &= -4 \end{cases}$ ?

a) $\begin{bmatrix} -1 \\ 3 \\ 0 \end{bmatrix}$

b) $\begin{bmatrix} 3 \\ 2 \\ 3 \end{bmatrix}$

c) $\begin{bmatrix} 1 \\ 0 \\ -5 \end{bmatrix}$

d) $\begin{bmatrix} 4 \\ 0 \\ 3 \end{bmatrix}$

11. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation with standard matrix $\begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$. Which of the following vectors belongs to the range of $T$?

a) $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$

b) $\begin{bmatrix} 10 \\ 10 \end{bmatrix}$

c) $\begin{bmatrix} -1 \\ -5 \end{bmatrix}$

d) $\begin{bmatrix} 5 \\ 10 \end{bmatrix}$

12. Find the standard matrix of the following linear transformation: $T(\mathbf{x})$ is the vector obtained from $\mathbf{x}$ by rotating it by 90° counterclockwise and multiplying its length by 5.

a) $\begin{bmatrix} 0 & -5 \\ 5 & 0 \end{bmatrix}$

b) $\begin{bmatrix} 5 & -5 \\ -5 & 5 \end{bmatrix}$

c) $\begin{bmatrix} 5 & 0 \\ 0 & -5 \end{bmatrix}$

d) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

13. Which of the following formulae defines a linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^2$?

a) $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1^2 + x_2 \\ x_1 - x_2 \\ x_3 \end{bmatrix}$

b) $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1 + x_2 \\ x_3 - x_1 \\ x_2 \end{bmatrix}$

c) $T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} x_1 x_2 \\ x_1 + x_2 + x_3 \end{bmatrix}$

d) $T\left(\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}\right) = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

14. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation such that $T(\begin{bmatrix} 1 \\ 1 \end{bmatrix}) = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$, and $T(\begin{bmatrix} 1 \\ -1 \end{bmatrix}) = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$. Find $T(\begin{bmatrix} 5 \\ 1 \end{bmatrix})$. (Hint: $\begin{bmatrix} 5 \\ 1 \end{bmatrix} = 3\begin{bmatrix} 1 \\ 1 \end{bmatrix} + 2\begin{bmatrix} 1 \\ -1 \end{bmatrix}$)

a) $\begin{bmatrix} 6 \\ 17 \end{bmatrix}$

b) $\begin{bmatrix} 2 \\ 5 \\ 0 \\ 1 \end{bmatrix}$

c) $\begin{bmatrix} 3 \\ 2 \end{bmatrix}$

d) $\begin{bmatrix} 6 \\ 15 \end{bmatrix}$

15. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation with the standard matrix $\begin{bmatrix} 1 & 0 & 5 \\ 0 & 2 & 1 \end{bmatrix}$, then which of the following statements is true?

a) $T$ is one-to-one and onto.

b) $T$ is one-to-one, but not onto.

c) $T$ is not one-to-one, but onto.

d) $T$ is not one-to-one, and not onto.