MATH 220 MIDTERM EXAMINATION October 19, 2004

Name____________________________ ID #_________________ Section #______

There are 20 multiple choice questions. Each problem is worth 5 points. Four possible answers are given for each problem, only one of which is correct. When you solve a problem, note the letter next to the answer that you wish to give and blacken the corresponding space on the answer sheet. **Mark only one choice; darken the circle completely** (you should not be able to see the letter after you have darkened the circle).

THE USE OF CALCULATORS DURING THE EXAMINATION IS FORBIDDEN.

PLEASE SHOW YOUR PSU ID CARD TO YOUR INSTRUCTOR WHEN YOU FINISH.
GOOD LUCK.

CHECK THE EXAMINATION BOOKLET BEFORE YOU START. THERE SHOULD BE 20 PROBLEMS ON 11 PAGES (INCLUDING THIS ONE).

1. Which of the following matrices is in **reduced** echelon form?

a) $A = \begin{bmatrix} 1 & 5 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$

b) $B = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

c) $C = \begin{bmatrix} 1 & 1 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

d) $D = \begin{bmatrix} 1 & 0 & 4 \\ 0 & 1 & 5 \\ 0 & 0 & 0 \end{bmatrix}$

2. Let $A = \begin{bmatrix} 2 & 0 & 4 & 6 \\ -1 & 1 & -3 & 2 \end{bmatrix}$. Find the general solution to the homogeneous equation $A\mathbf{x} = 0$.

a) $\begin{cases} x_1 = -2x_3 - 3x_4 \\ x_2 = x_3 - 5x_4 \\ x_3,\ x_4 \text{ are free} \end{cases}$

b) $\begin{cases} x_1 = 2x_3 - 3x_4 \\ x_2 = 6x_3 - 5x_4 \\ x_3,\ x_4 \text{ are free} \end{cases}$

c) $\begin{cases} x_1 = x_3 - x_4 \\ x_2 = x_3 + x_4 \\ x_3,\ x_4 \text{ are free} \end{cases}$

d) $\begin{cases} x_1 = 7x_4 \\ x_2 = x_3 - x_4 \\ x_3,\ x_4 \text{ are free} \end{cases}$

3. Consider the linear system: $\begin{cases} x_1 + x_2 - x_3 = 2 \\ 4x_1 + 3x_2 = 5. \end{cases}$ Find the parametric vector form of its solution set.

   a) $\mathbf{x} = \begin{bmatrix} -1 \\ 3 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} -3 \\ 4 \\ 1 \end{bmatrix}$

   b) $\mathbf{x} = \begin{bmatrix} -1 \\ 3 \end{bmatrix} + x_3 \begin{bmatrix} 3 \\ -4 \end{bmatrix}$

   c) $\mathbf{x} = \begin{bmatrix} 2 \\ 5 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 3 \\ -4 \\ 1 \end{bmatrix}$

   d) $\mathbf{x} = \begin{bmatrix} -1 \\ -3 \\ 0 \end{bmatrix} + x_3 \begin{bmatrix} 3 \\ 4 \\ 1 \end{bmatrix}$

4. Consider the following vectors $\mathbf{v}_1 = \begin{bmatrix} 1 \\ -1 \\ 2 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}$ $\mathbf{v}_3 = \begin{bmatrix} -1 \\ 3 \\ h \end{bmatrix}$. For what value(s) of h is the following set $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ linearly **dependent**?

   a) $h = 4$

   b) $h = -4$

   c) All real numbers $h \neq 0$

   d) $h = 0$

5. Which of the following vectors is NOT in $\text{Span}\left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \right\}$?

a) $\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$

b) $\begin{bmatrix} 10 \\ 5 \\ 0 \end{bmatrix}$

c) $\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$

d) $\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

6. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation that first rotates vectors of $\mathbb{R}^2$ counterclockwise through 90° about the origin, then reflects vectors about the line $y = -x$. Find the standard matrix of T.

a) $\begin{bmatrix} 1 & -1 \\ 1 & 0 \end{bmatrix}$

b) $\begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$

c) $\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

d) $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$

7. Let $\mathbf{u} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $\mathbf{v} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, and let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation such that $T(\mathbf{u}) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $T(\mathbf{v}) = \begin{bmatrix} 2 \\ 5 \end{bmatrix}$. Find the image of $3\mathbf{u} - \mathbf{v}$ under the transformation T.

a) $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

b) $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$

c) $\begin{bmatrix} 1 \\ 7 \end{bmatrix}$

d) $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$

8. If T is a linear transformation whose standard matrix is given by $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & 0 \\ -1 & 2 & 1 \end{bmatrix}$, then which of the following statements is true?

a) T is one-to-one, but not onto.

b) T is not one-to-one, but it is onto

c) T is both one-to-one and onto

d) T is neither one-to-one nor onto

9. Which of the following set of vectors is linearly independent?

a) $\left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 5 \\ 0 \end{bmatrix}, \begin{bmatrix} 4 \\ 9 \end{bmatrix} \right\}$

b) $\left\{ \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}, \begin{bmatrix} 2 \\ 6 \\ 10 \end{bmatrix} \right\}$

c) $\left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix} \right\}$

d) $\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \right\}.$

10. Let $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & -1 \\ 1 & 2 & 1 \end{bmatrix}$. How many rows of A contain a pivot position?

a) 1

b) 2

c) 3

d) 4

11. Let $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & 5 \\ 0 & 1 & 1 \end{bmatrix}$. The solution set of the homogeneous equation $A\mathbf{x} = 0$ is

a) Only the trivial solution $\mathbf{x} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$

b) A line through the origin.

c) A plane through the origin.

d) All $\mathbb{R}^3$.

12. Let $T : \mathbb{R}^5 \to \mathbb{R}^2$ be a linear transformation. Which of the following statements is **always true** for such transformations.

a) T is onto

b) T is one-to-one

c) The standard matrix of T is a 2×5 matrix

d) The standard matrix of T is a 5×2 matrix

13. Which of the following transformations is linear?

a) $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} 4 \\ x_1 - x_2 \end{bmatrix}$

b) $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} x_1^3 - x_2 \\ x_1 - x_2 \end{bmatrix}$

c) $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} 3x_1 \\ x_1 - 5x_2 \end{bmatrix}$

d) $T\left(\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}\right) = \begin{bmatrix} \sin(x_1) \\ x_1 - x_2 \end{bmatrix}$

14. The inverse of the matrix $A = \begin{bmatrix} 5 & 7 \\ 2 & 3 \end{bmatrix}$ is

a) $\begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}$

b) $\begin{bmatrix} -3 & 7 \\ -2 & 5 \end{bmatrix}$

c) $\begin{bmatrix} 3 & -7 \\ -2 & 5 \end{bmatrix}$

d) $\begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}$

15. If $A = \begin{bmatrix} 2 & 0 & 1 \\ 3 & 1 & 5 \end{bmatrix}$ and $B == \begin{bmatrix} 1 & 0 \\ 4 & -1 \\ 2 & 0 \end{bmatrix}$. What is BA?

a) $\begin{bmatrix} 4 & 0 \\ 17 & -1 \end{bmatrix}$

b) $\begin{bmatrix} 2 & 0 & 1 \\ 5 & -1 & -1 \\ 4 & 0 & 2 \end{bmatrix}$

c) $\begin{bmatrix} 2 & 1 \\ 5 & 1 \end{bmatrix}$

d) $\begin{bmatrix} 2 & 0 & 3 \\ 5 & 1 & 1 \\ 0 & 0 & 2 \end{bmatrix}$

16. If $A = \begin{bmatrix} 2 & 1 & 1 \\ 4 & -6 & 0 \\ -2 & 7 & 2 \end{bmatrix}$, then $A\mathbf{x} = 0$ has

a) only one solution

b) two solutions

c) infinitely many solution

d) No solution.

17. Suppose A is a 4×5 matrix such that each row contains a pivot. Which of the following statements is **false**?

a) The columns of A span $\mathbb{R}^4$.

b) $Ax = 0$ has a free variable

c) The columns of A are linearly independent

d) The columns of A are linearly dependent

18. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation. If $T(\mathbf{e}_1) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ and $T(2\mathbf{e}_1 + \mathbf{e}_2) = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$, what is the standard matrix of T?

a) $\begin{bmatrix} 1 & 3 \\ 2 & 5 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}$

c) $\begin{bmatrix} 1 & -1 \\ 2 & 0 \end{bmatrix}$

d) $\begin{bmatrix} 3 & -1 \\ 2 & 4 \end{bmatrix}$

19. Let $A = \begin{bmatrix} 1 & 0 & 4 \\ -2 & 1 & -6 \\ 1 & -3 & -2 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$. Then $A\mathbf{x} = \mathbf{b}$ is **consistent** if

a) $b_1 + b_2 + b_3 = 0$

b) $2b_1 - b_2 + b_3 = 0$

c) $5b_1 + 3b_2 + b_3 = 0$

d) $b_1 - b_2 + b_3 = 0$

20. Which of the following statements is **false**?

a) Every homogeneous linear system is consistent.

b) If A is a 3×2 then the linear transformation defined by $T(\mathbf{x}) = A\mathbf{x}$ cannot be onto

c) If A is a 2×3 then the linear transformation defined by $T(\mathbf{x}) = A\mathbf{x}$ cannot be onto

d) Two nonzero vectors in $\mathbb{R}^n$ are linearly dependent if one of the vectors is a scalar multiple of the other.