

1. If

$$\begin{bmatrix} 1 & 2 & -3 \\ 5 & h & 4 \end{bmatrix}$$

is the augmented matrix of a system of linear equations, then for what value of h is the system inconsistent?

a) $h = 0$.

b) $h = 5$.

c) $h = 10$.

d) $h = 15$.

$$\begin{bmatrix} 1 & 2 & -3 \\ 5 & h & 4 \end{bmatrix} \xrightarrow{-5 \times \text{row 1} + \text{row 2}} \sim \begin{bmatrix} 1 & 2 & -3 \\ 0 & h-10 & 19 \end{bmatrix}$$

2. Which of the following matrices is in row echelon (but not necessarily reduced row echelon) form?

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 2 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

a) A and C , but not B .

b) A and B , but not C .

c) B only.

d) B and C , but not A .

3. If $A = \begin{bmatrix} 1 & 1 & 2 & 8 \\ -1 & -2 & 3 & 1 \\ 3 & -7 & 4 & 10 \end{bmatrix}$, then find the matrix in reduced row echelon form that is row equivalent to A .

a) $\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$

c) $\begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 3 \end{bmatrix}$

d) $\begin{bmatrix} 1 & 0 & 0 & 5 \\ 0 & 1 & 0 & 7 \\ 0 & 0 & 0 & 6 \end{bmatrix}$

$\sim \begin{bmatrix} 1 & 1 & 2 & 8 \\ -1 & -2 & 3 & 1 \\ 3 & -7 & 4 & 10 \end{bmatrix} \quad \sim \begin{bmatrix} 1 & 1 & 2 & 8 \\ 0 & -1 & 5 & 9 \\ 0 & -10 & -2 & -14 \end{bmatrix} \quad \begin{array}{l} 1 \times \textcircled{1} + \textcircled{2} \\ -3 \times \textcircled{1} + \textcircled{3} \end{array} \quad \sim \begin{bmatrix} 1 & 1 & 0 & 4 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$

$\sim \begin{bmatrix} 1 & 1 & 2 & 8 \\ 0 & -1 & 5 & 9 \\ 0 & -10 & -2 & -14 \end{bmatrix} \quad \begin{array}{l} -1 \times \textcircled{2} \\ -10 \times \textcircled{2} + \textcircled{3} \end{array} \quad \sim \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$

$\sim \begin{bmatrix} 1 & 1 & 2 & 8 \\ 0 & 1 & -5 & -9 \\ 0 & 0 & -52 & -14 \end{bmatrix} \quad \begin{array}{l} -1 \times \textcircled{2} \\ -\frac{1}{52} \times \textcircled{3} \end{array}$

4. If $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}$ and $\mathbf{v}_2 = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}$, then for what value of h does the vector $\mathbf{v}_3 = \begin{bmatrix} 7 \\ 5 \\ h \end{bmatrix}$ lie in the plane generated by \mathbf{v}_1 and \mathbf{v}_2 ?

a) $h = 4$.

b) $h = -4$.

c) $h = 8$.

d) $h = -8$.

If $x_1 \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix} + x_2 \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix} = \begin{bmatrix} 7 \\ 5 \\ h \end{bmatrix}$ has solution,

then \vec{v}_3 is in the plane of \vec{v}_1 and \vec{v}_2 .

So $\begin{bmatrix} 1 & 2 & 7 \\ 1 & 1 & 5 \\ -2 & -1 & h \end{bmatrix} \sim \begin{bmatrix} 1 & 2 & 7 \\ 0 & -1 & -2 \\ 0 & 3 & 14+h \end{bmatrix}$

$\sim \begin{bmatrix} 1 & 2 & 7 \\ 0 & -1 & -2 \\ 0 & 0 & 8+h \end{bmatrix} \quad 8+h = 0$

5. What is the solution set of the following system of linear equations?

$$\begin{aligned} -2x_2 + 3x_3 &= 1 \\ 2x_1 + 4x_2 - x_3 &= 2 \\ -6x_1 - 12x_2 + 3x_3 &= 5 \end{aligned}$$

$$\begin{bmatrix} 0 & -2 & 3 & 1 \\ 2 & 4 & -1 & 2 \\ -6 & -12 & 3 & 5 \end{bmatrix}$$

a) $x_1 = 1, x_2 = 0, x_3 = -1.$

b) $x_1 = 3 - 2t, x_2 = 2 + t, x_3 = t$

c) $x_1 = 2 + 2t, x_2 = t, x_3 = 3.$

$$\sim \begin{bmatrix} \textcircled{1} & 2 & -\frac{1}{2} & 1 \\ 0 & \textcircled{-2} & 3 & 1 \\ 0 & 0 & 0 & \textcircled{11} \end{bmatrix}$$

d) There are no solutions. The system is inconsistent.

6. If $A = \begin{bmatrix} 1 & 5 & -2 & 3 \\ 2 & -3 & 1 & 4 \\ 0 & 2 & 4 & 6 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 2 \\ -1 \\ 6 \\ -3 \end{bmatrix}$, then what is the second entry of $A\mathbf{v}$?

a) 1.

b) 2.

c) 3.

d) 4.

$$\begin{bmatrix} 2 & -3 & 1 & 4 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ 6 \\ -3 \end{bmatrix} = 4 + 3 + 6 - 12$$

7. If $A = \begin{bmatrix} 1 & -6 & -3 \\ 2 & 12 & 2 \\ -1 & -3 & 0 \end{bmatrix}$, then what geometric figure is formed from the span of the columns of A ?

- a) A line.
 (b) A plane.
 c) All of \mathbb{R}^3 .
 d) All of \mathbb{R}^2 .

$$\begin{bmatrix} 1 & -6 & -3 \\ 2 & 12 & 2 \\ -1 & -3 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & -6 & -3 \\ 0 & 24 & 8 \\ 0 & -9 & -3 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & -6 & -3 \\ 0 & 3 & 1 \\ 0 & -9 & -3 \end{bmatrix}$$

$$\sim \begin{bmatrix} \textcircled{1} & -6 & -3 \\ 0 & \textcircled{3} & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

8. If A is a 4 by 4 matrix whose columns span \mathbb{R}^4 , then which of the following statements is false?

- a) The equation $Ax = b$ has at least one solution for every b . ✓
 b) The columns of A are linearly independent. ✓
 (c) The equation $Ax = 0$ has a nontrivial solution. ✗ *Only trivial solution*
 d) The linear transformation $x \mapsto Ax$ is onto. ✓

9. If $A = \begin{bmatrix} 4 & -8 \\ -3 & 6 \\ 2 & -4 \end{bmatrix}$ is the coefficient matrix of a system of linear equations, then what is the solution set for the associated homogeneous linear system?

a) $x_1 = 1, x_2 = 2.$

b) $x_1 = 0, x_2 = 0.$

c) $x_1 = 2x_2, x_2 = t.$

d) There are no solutions because the the system is inconsistent.

$$\begin{bmatrix} 4 & -8 & 0 \\ -3 & 6 & 0 \\ 2 & -4 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & -2 & 0 \\ 1 & -2 & 0 \\ 1 & -2 & 0 \end{bmatrix} \sim \begin{bmatrix} \textcircled{1} & -2 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

10. If T is the linear transformation defined by the formula

$$T(x_1, x_2) = (x_2, -x_1, x_1 + 3x_2, x_1 - x_2),$$

then what is the standard matrix for T ?

a) $\begin{bmatrix} 0 & -1 & 1 & 1 \\ 1 & 0 & 3 & -1 \end{bmatrix}$

b) $\begin{bmatrix} 1 & 0 & 3 & -1 \\ 0 & -1 & 1 & 1 \end{bmatrix}$

c) $\begin{bmatrix} 0 & 1 \\ -1 & 0 \\ 1 & 3 \\ 1 & -1 \end{bmatrix}$

d) $\begin{bmatrix} -1 & 0 \\ 0 & 1 \\ 3 & -1 \\ -1 & 1 \end{bmatrix}$

$$A = [T(\vec{e}_1), T(\vec{e}_2)]$$

$$= \begin{bmatrix} 0 & 1 \\ -1 & 0 \\ 1 & 3 \\ 1 & -1 \end{bmatrix}$$

11. If $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$ is the linear transformation that first rotates points clockwise by $\pi/4$ about the origin and then reflects points about the x_1 -axis, then what is the standard matrix for T ?

a) $\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & -1 \\ -1 & -1 \end{bmatrix}$.

b) $\frac{1}{\sqrt{2}} \begin{bmatrix} -1 & 1 \\ 1 & 1 \end{bmatrix}$.

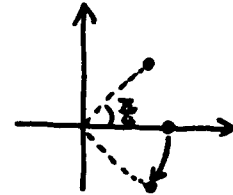
c) $\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$.

d) $\frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$.

$$A = [T(\vec{e}_1) , T(\vec{e}_2)]$$

$$T(\vec{e}_1) = \begin{bmatrix} \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} \end{bmatrix}$$

$$T(\vec{e}_2) = \begin{bmatrix} \frac{\sqrt{2}}{2} \\ -\frac{\sqrt{2}}{2} \end{bmatrix}$$



12. If

$$A = \begin{bmatrix} 5 & 2 \\ 1 & 4 \\ 3 & 0 \end{bmatrix} \text{ and } B = \begin{bmatrix} -1 & 2 \\ 3 & -1 \end{bmatrix},$$

then what is AB ?

a) $\begin{bmatrix} 1 & 8 \\ 11 & -2 \\ -3 & 6 \end{bmatrix}$.

b) $\begin{bmatrix} 1 & 8 \\ 2 & 2 \\ -3 & 6 \end{bmatrix}$.

c) $\begin{bmatrix} -1 & -8 \\ 5 & 0 \\ 16 & 3 \end{bmatrix}$.

d) $\begin{bmatrix} -1 & -8 \\ 3 & 4 \\ 3 & -6 \end{bmatrix}$.

13. If $A = \begin{bmatrix} 2 & 3 \\ 0 & 4 \end{bmatrix}$ and If $B = \begin{bmatrix} 2 & 1 \\ 0 & h \end{bmatrix}$, then for what value of h does $AB = BA$?

a) $h = 1$.

b) $h = 2$.

c) $h = 8/3$.

d) $h = 3/8$.

$$AB = \begin{bmatrix} 4 & 2+3h \\ 0 & 4h \end{bmatrix}$$

$$BA = \begin{bmatrix} 4 & 10 \\ 0 & 4h \end{bmatrix}$$

$$AB = BA \Rightarrow 2+3h = 10.$$

14. If $A = \begin{bmatrix} 8 & 6 \\ -9 & -7 \end{bmatrix}$, then what is the inverse of A ?

a) $\begin{bmatrix} -7 & -6 \\ 9 & 8 \end{bmatrix}$.

b) $-\frac{1}{2} \begin{bmatrix} -7 & -6 \\ 9 & 8 \end{bmatrix}$.

c) $\frac{1}{2} \begin{bmatrix} -7 & -6 \\ 9 & 8 \end{bmatrix}$.

d) $\begin{bmatrix} 7 & 6 \\ 9 & 8 \end{bmatrix}$.

15. If $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$, then what is the inverse of A ?

a) $\begin{bmatrix} 1/2 & 0 & 0 \\ 0 & -1 & 0 \\ -1/2 & 1 & 1 \end{bmatrix}$.

b) $\begin{bmatrix} 1/2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$.

c) $\begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 1 & 0 \\ 1/2 & 1 & 1 \end{bmatrix}$.

d) $\begin{bmatrix} -1/2 & 0 & 0 \\ 0 & 1 & 0 \\ 1/2 & 1 & 1 \end{bmatrix}$.

16. If $A = \begin{bmatrix} -1 & 1 \\ 2 & 1 \end{bmatrix}$, then what is A^3 ?

a) $\begin{bmatrix} -1 & 1 \\ 8 & -1 \end{bmatrix}$.

b) $\begin{bmatrix} 7 & 5 \\ 10 & -7 \end{bmatrix}$.

c) $\begin{bmatrix} -3 & 3 \\ 6 & 3 \end{bmatrix}$.

d) $\begin{bmatrix} 1 & -1 \\ -8 & 1 \end{bmatrix}$.

17. Which of the following statements is true?

- a) If A is an invertible matrix, then the columns of A are linearly independent. ✓
- b) If A is an n by n matrix that has n pivot positions, then the equation $Ax = 0$ has a nontrivial solution. X Only trivial solution.
- c) If A^T is not invertible, A is not invertible. ✓
- d) If A is an n by n matrix, then the linear transformation $x \mapsto Ax$ is one-to-one, then A is invertible. X $A\vec{x} = \vec{b}$ might not have a solution.

18. If $(A^T)^{-1} = \begin{bmatrix} -3 & -1 \\ 5 & 2 \end{bmatrix}$, then what is A ?

a) $\begin{bmatrix} 2/5 & -1 \\ 1/5 & -3/5 \end{bmatrix}$.

b) $\begin{bmatrix} -2 & 5 \\ -1 & 3 \end{bmatrix}$.

c) $\begin{bmatrix} 2/5 & 1/5 \\ -1 & -3/5 \end{bmatrix}$.

d) $\begin{bmatrix} -3 & 5 \\ -1 & 2 \end{bmatrix}$.

$$A^T = \begin{bmatrix} -2 & -1 \\ 5 & 3 \end{bmatrix}$$

$$A = \begin{bmatrix} -2 & 5 \\ -1 & 3 \end{bmatrix}$$

