

Homework 2

Math 251, Section 3

due Mon Sep 8, 2008

- 1. integrating factor. (5P)** Solve the following differential equations!
 - (a) $\dot{y} + \frac{1}{2}y = \sin t$,
 - (b) $y' + \frac{2}{x}y = 5$ with $y(1) = 0$,
 - (c) $t\dot{y} + y = \sin t$, $t > 0$
 - (d) $y' + xy = e^{-\frac{1}{2}x^2}$
- 2. separation of variables. (5P)** Solve the following differential equations and determine the behaviour for big x !
 - a) $y' + y^2(\cos x + 1) = 0$ for $x > 0$,
 - b*) $y' = \frac{x - e^x}{y + e^y}$,
 - c) $\dot{y} = t(4 - y)/(1 + t)$, $y(0) = y_0 > 0$
- 3. Vocabulary. (5P)** A student starts learning Chinese. He learns 10 characters per week but at the same time also forgets 5% of his vocabulary per week.
 - a) Draw the number of his known vocabulary over the weeks.
 - b) How long does it take him to learn 100 vocables?
 - c) It is said that one needs 3000 vocables to read a newspaper. Will he ever be able to read one?
- 4. Cold beer. (5P)** Suppose you put 3 bottles (1kg) of cold beer (34°F) into a cooler without ice. Assume the outside temperature is 100°F. The cooler lets in heat proportional to the current temperature difference, with a factor of $6 \frac{\text{J}}{\text{°F}\cdot\text{min}}$. The temperature of the beer rises because of the absorbed heat with a factor of $2000 \frac{\text{J}}{\text{kg}\cdot\text{°F}}$.
 - a) Set up a differential equation that describes the temperature of the beer over the time. Check the units carefully.
 - b) Draw the temperature over the time. How long is the beer drinkable, i.e. the temperature below 40°F?
Hint: check the units carefully. To avoid using a calculator you can use $\ln(1 - x) \approx -x$, for small x in the last step.