Analysis A
Measure Theory

Description

Metric spaces. Continuity. Topological Spaces. Sets of measure zero. Lebesgue measure. Integration. Differentiation. Lebesgue spaces. Other topics as time permits.

Course Objectives

The centerpiece of the course is Lebesgue integration theory. The course also covers basic notions and results about abstract topological and metric spaces, as well as some applications of Lebesgue theory.

Syllabus

1. **Metric spaces.** Metrics. Open and closed sets. Examples, including Cantor sets. Convergence and completeness. Subspaces. Baire category.

2. **Continuity.** Continuous functions and homeomorphisms. Uniform continuity. Contraction mapping principle.

3. **Topological spaces.** Basic properties and examples. Connectedness, path connectedness and components.

4. **Compact Spaces.** Sequential compactness and covering compactness. Lebesgue covering theorem. Product of compact spaces. Locally compact spaces. The Stone-Weierstrass theorem.

5. **Sets of measure zero.** Sets of Lebesgue measure zero in Euclidean space. Examples and non-examples: countable sets, the Cantor set, intervals. Uncountability of $\mathbb{R}$ and an informal discussion of cardinality. Cardinality of the Cantor set.

6. **Lebesgue measure.** Construction of Lebesgue measure on $\mathbb{R}^n$. Outer measure, measurable sets and Lebesgue measure. Non-measurable sets. Additivity and continuity properties. Approximation by open and closed sets. Measurable functions. Lusin's theorem.

7. **Integration.** Construction of the Lebesgue integral. Basic properties. Fatou's lemma. Monotone convergence theorem. Dominated convergence theorem.

8. **Differentiation.** Differentiation of monotone functions. Functions of bounded variation. Differentiating under the integral sign. Absolute continuity. The Radon-Nikodim theorem.

9. L^p **spaces.** Measure spaces and measurable functions. The spaces L^1 and L^2, including completeness. Fubini's theorem on $\mathbb{R}^n$. Brief discussion of product measures and the general case. Convolutions and the smoothing properties of convolution.

10. **Other optional topics, as time permits.** Transfinite hierarchy and induction. Fourier transform, inversion formula and Plancherel theorem. Hausdorff measure and Hausdorff dimension, including computations for Cantor sets. Fractal sets.

Other Information

Recommended text is H. L. Royden, *Real Analysis*, 2nd edition, Stanford University, the Macmillan Company.