Algebra B
Abstract Algebra

Description

Groups. Rings. Field extensions. Galois theory. Other topics as time permits.

Course objectives

This course aims to provide the grounding necessary study advanced areas of abstract algebra. The nineteenth-century triumphs of Galois theory given here remain beautiful and inspiring.

Syllabus

1. **Groups.** Subgroups. Cosets. Normal subgroups. Quotient groups. Group homomorphisms. Examples: symmetric groups, alternating groups, dihedral groups, general linear groups, special linear groups, orthogonal groups, and unitary groups. Homomorphism theorems. Cyclic groups.

2. **Group actions** Orbits. Stabilizers. Sylow theorems andelementary consequences for simple or non-abelian groups of small order.

3. **Further concepts.** Direct products. Direct sums. Semi-direct products. Nilpotent and solvable groups.

4. **Rings.** Ideals. Quotient rings. Ring homomorphisms. Homomorphism theorems. Prime ideals. Maximal ideals. Polynomial rings. Integral domains and fields of fractions. Unique factorization domains. Euclidean domains. Principal ideal domains. Gauss's lemma. Eisenstein criterion.

5. **Finitely generated modules over principal ideal domains** (If not done in Algebra A.)

6. **Field extensions** Algebraic and transcendental elements. Degree of an extension. Degree of an (algebraic) element. Simple field extensions. Algebraic numbers. Construction and uniqueness of field extensions up to Isomorphism. Splitting fields. Algebraic closure

7. **Finite fields.** Existence and uniqueness.

8. **Separability.** Separable polyomials. Separable extensions. Embeddings into a fixed algebraic closure. Primitive element theorem.

9. **Galois theory.** Normal extensions. Solvability of Galois group versus solvability of a polynomial by radicals (in characteristic zero). Constructions by straight-edge and compass. Cyclotomic extensions. Galois groups of cubic polynomials.

10. **Other optional topics, as time permits.** General structure of fields. Transcendence basis. Transcendence degree. Inseparable extensions.

Other Information

Recommended text is:

[1] A.W. Knapp, *Basic Algebra*, Birkhäuser, Boston, 2006.

Other sources are:

[2] N. Jacobson, *Basic Algebra*, 2 vols, 2nd ed. J.W.H. Freeman & Co, San Francisco, 1985 & 1989.

[3] S. Lang, *Algebra*, Addison-Wesley, Reading, MA, 3rd ed. Springer-Verlag, New York, 2002.

[4] J.J. Rotman, *Algebra*, Prentice-Hall, 2002. B.L. van der Waerden, *Modern Algebra*, various translated editions.