Algebra A
Linear Algebra

Description

Vector spaces. Linear transformations. Bilinear forms. Canonical forms for linear transformations. Multilinear algebra. Other topics as time permits.

Course Objectives

This course reviews undergraduate linear algebra and proceeds to more advanced topics. Its purpose is to provide a solid understanding of linear algebra of the sort needed throughout graduate mathematics.

Syllabus

1. **Vector spaces.** Fields. Vector spaces. Subspaces. Spanning sets. Linearly independent sets. Bases. Dimension.

2. **Linear transformations.** Kernel and image. Matrices. Direct sums and quotients. Correspondence between linear transformations and matrices. Matrix operations. Standard homomorphism theorems. Rank-nullity theorem.

3. **Inner products and quadratic forms.** Bilinear functions. Sesquilinear functions. Orthogonal sets and orthonormal sets. Norm. Schwarz's inequality. Bessel's inequality. Adjoints. Self-adjoint, normal and unitary matrices. Orthogonal projections and orthogonal complements. Gram-Schmidt orthogonalization process. Parseval's equality. Diagonalizability of normal operators. Quadratic forms over $\mathbb{R}$. Signature. Sylvester's law. Positive (semi)definiteness. Spectral Theorem.

4. **Theory of a single endomorphism of a finite-dimensional vector space.** Determinants. Cramer's rule. Multiplicative property of the determinant. Minimal and characteristic polynomial. Cayley-Hamilton Theorem. Modules over a principal ideal domain. Primary decomposition. Application to finitely generated abelian groups. If F is a field, $\mathsf{F}[\mathsf{x}]$ is a principal ideal domain. Rational and Jordan canonical forms.

5. **Multilinear algebra.** Symmetric and alternating bilinear forms. Groups leaving a bilinear form invariant. Tensor products of vector spaces. Tensor algebra. Identification of tensor algebra with free associative algebra. Symmetric algebra. Exterior algebra.