Geometry and Topology
Qualifying Exam

May 11, 1994

Let $\mathbb{R}^n$ denote n-dimensional euclidean space; let S^n denote the unit sphere in R^{n+1}; and let RP_n denote n-dimensional real projective space. Give complete proofs or justifications for all your statements.

Do six of the following eight problems.

1. Prove that every compact Hausdorff space embeds in the product of some number of copies of $[0, 1]$.

2. There exists a countable and connected Hausdorff space X containing more than one point. Show that X cannot be metrizable.

3. (a) Compute $\pi(RP_2)$.
   (b) Use induction and van Kampen's Theorem to compute $\pi(RP_m)$ for all $n > 2$.

4. Suppose that Let $f : S^2 \to S^1 X S^1$ is a continuous function. Show that f is homotopic to a constant map. (A constant map is one carrying every point of the domain to a single point of the range.)

5. Using the Mayer-Vietoris exact sequence, compute the singular homology groups, with integer coefficients, of RP_2 and RP_3.

6. Let A and B be subsets of S^4 homeomorphic to S^1 and S^2 respectively, with $A \cap B$ a single point. Find the singular homology groups of $S^4 - A \cup B$.

7. Let $f : S^2 \to R^4$ be a smooth map which does not contain the origin. Show that there is a line through the origin in R^4 is disjoint from $f : (S^2)$.

8. Exhibit a 1-form without zero on the circle S^1. Show that such a form cannot be exact.